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About Graph Complements

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Summary. This article formalizes different variants of the complement graph in the Mizar system [3], based on the formalization of graphs in [6].

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0. Introduction

In the first section of this article, the property of a graph to be reflexive is rigorously introduced. But since the irreflexive attribute was called `loopless` in [6], `loopfull` was chosen this time.

The following section introduces a mode to add loops to a subset of the vertices of a graph. It is shown that for a finite subset this operation can be done by adding a loop at a time (cf. [5]). It is also shown that adding loops can preserve isomorphism between graphs, if the subset of vertices of the second graph the loops are added to is the image under an isomorphism of the subset of vertices of the first graphs the loops are added to.

The next four sections formalize the directed complement with loops, the undirected complement with loops, the directed complement without loops and the undirected complement without loops, respectively. Given a simple undirected graph, its complement is usually defined on the same vertex set; two different vertices being adjacent iff they weren't adjacent in the original graph

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[8], [2], [1]. A similar definition can be given for simple digraphs [1]. The loop variants are introduced on the base of similarity between graphs and relations.

In contrast to the literature the definitions formalized allow to take the complement of any graphs, with parallel edges simply being ignored. So any complement of a graph is also a complement of that graph with its parallel edges removed. Furthermore on a technical note, the vertex sets of the graph and its complement are required to be the same, while the edge sets have to be disjoint. This choice was made to ensure the union of a graph and its complement would be complete and its intersection edgeless. Since the edge set of the complement graph is otherwise unspecified, for each complement type all possible complements of a graph are only isomorphic to each other. Other theorems include:

- Involutiveness of the graph complement: If a graph is of the right type (e.g. simple for undirected complement without loops), then it is the complement of its complement.
- The complement of an edgeless graph is complete.
- The undirected complement without loops of a complete graph is edgeless.
- The complement of an unconnected graph is connected.
- The neighbors of a vertex in a complement without loops of a graph is the complement of the neighbors in the original graph.
- If a graph has order at least 3, no vertex can be an endvertex in both that graph and its directed complement without loops (the directed K_2 is a counterexample for order equal to 2.)
- If a graph has order at least 4, no vertex can be an endvertex in both that graph and its undirected complement without loops (P_3 with its complement $K_2 + K_1$ is a counterexample for order equal to 3.)

The last section briefly introduces the property of a graph to be self-complementary for all four variants, but without going into depth. However, it is shown that these four variants are mutually exclusive, except for K_1 which is self-complementary with respect to the directed or undirected complement, without loops in both cases.

1. Loopfull Graphs

Let G be a graph. We say that G is loopfull if and only if

(Def. 1) for every vertex v of G, there exists an object e such that e joins v and v in G.

Let us consider a graph G. Now we state the propositions:

(1) G is loopfull if and only if for every vertex v of G, there exists an object e such that e joins v to v in G.

(2) G is loopfull if and only if for every vertex v of G, v and v are adjacent.

One can verify that every graph which is loopfull is also non loopless and every graph which is trivial and non loopless is also loopfull and there exists a graph which is loopfull and complete and there exists a graph which is non loopfull.

Now we state the proposition:

(3) Let us consider a graph G_1, a set E, and a graph G_2 given by reversing directions of the edges E of G_1. Then G_1 is loopfull if and only if G_2 is loopfull.

Let G be a loopfull graph and E be a set. One can check that every graph given by reversing directions of the edges E of G is loopfull.

Let G be a non loopfull graph. Let us observe that every graph given by reversing directions of the edges E of G is non loopfull.

Now we state the propositions:

(4) Let us consider graphs G_1, G_2. If $G_1 \approx G_2$, then if G_1 is loopfull, then G_2 is loopfull.

(5) Let us consider a loopfull graph G_2, and a supergraph G_1 of G_2. Suppose the vertices of $G_1 =$ the vertices of G_2. Then G_1 is loopfull.

Let us consider graphs G_1, G_2 and a partial graph mapping F from G_1 to G_2. Now we state the propositions:

(6) Suppose $\operatorname{rng} F_{\mathbb{V}} =$ the vertices of G_2 and G_1.loops() $\subseteq \operatorname{dom}(F_{\mathbb{E}})$. Then if G_1 is loopfull, then G_2 is loopfull.

(7) If F is total and onto, then if G_1 is loopfull, then G_2 is loopfull. The theorem is a consequence of (6).

(8) Suppose F is semi-continuous and $\operatorname{dom}(F_{\mathbb{V}}) =$ the vertices of G_1 and G_2.loops() $\subseteq \operatorname{rng} F_{\mathbb{E}}$. Then if G_2 is loopfull, then G_1 is loopfull.

(9) If F is total, onto, and semi-continuous, then if G_2 is loopfull, then G_1 is loopfull. The theorem is a consequence of (8).

(10) If F is isomorphism, then G_1 is loopfull iff G_2 is loopfull.

Let G be a loopfull graph and V be a set. Let us observe that every subgraph of G induced by V is loopfull and every subgraph of G with vertices V removed is loopfull and every subgraph of G with vertex V removed is loopfull.

Let G be a non loopfull graph. Let us observe that every spanning subgraph of G is non loopfull.

Let E be a set. Let us note that every subgraph of G induced by the vertices of G and E is non loopfull and every subgraph of G with edges E removed is non loopfull and every subgraph of G with edge E removed is non loopfull.

Now we state the proposition:

(11) Let us consider a graph G_2, a set V, and a supergraph G_1 of G_2 extended by the vertices from V. Suppose $V \setminus$ (the vertices of G_2) $\neq \emptyset$. Then G_1 is not loopfull.

Let G be a non loopfull graph and V be a set. Observe that every supergraph of G extended by the vertices from V is non loopfull.

Let G be a loopfull graph and v, e, w be objects. One can verify that every supergraph of G extended by e between vertices v and w is loopfull.

Now we state the propositions:

(12) Let us consider a graph G_2, a vertex v of G_2, objects e, w, and a supergraph G_1 of G_2 extended by v, w and e between them. Suppose $e \notin$ the edges of G_2 and $w \notin$ the vertices of G_2. Then G_1 is not loopfull.

(13) Let us consider a graph G_2, objects v, e, a vertex w of G_2, and a supergraph G_1 of G_2 extended by v, w and e between them. Suppose $e \notin$ the edges of G_2 and $v \notin$ the vertices of G_2. Then G_1 is not loopfull.

Let G be a non loopfull graph and v, e, w be objects. Let us observe that every supergraph of G extended by v, w and e between them is non loopfull.

Now we state the proposition:

(14) Let us consider a graph G_2, an object v, a subset V of the vertices of G_2, and a supergraph G_1 of G_2 extended by vertex v and edges between v and V of G_2. Suppose $v \notin$ the vertices of G_2. Then G_1 is not loopfull.

Let G be a non loopfull graph, v be an object, and V be a set. One can check that every supergraph of G extended by vertex v and edges between v and V of G is non loopfull.

Let G be a loopfull graph. Let us note that every subgraph of G with parallel edges removed is loopfull and every subgraph of G with directed-parallel edges removed is loopfull.

Let G be a non loopfull graph. Note that every subgraph of G with parallel edges removed is non loopfull and every subgraph of G with directed-parallel edges removed is non loopfull.

Let G_F be a graph-yielding function. We say that G_F is loopfull if and only if

(Def. 2) for every object x such that $x \in \operatorname{dom} G_F$ there exists a graph G such that $G_F(x) = G$ and G is loopfull.

Let G be a loopfull graph. Let us note that $\langle G \rangle$ is loopfull and $\mathbb{N} \longmapsto G$ is loopfull.

Let G_F be a non empty, graph-yielding function. Note that G_F is loopfull if and only if the condition (Def. 3) is satisfied.

(Def. 3) for every element x of $\operatorname{dom} G_F$, $G_F(x)$ is loopfull.

Let G_{Sq} be a graph sequence. Let us note that G_{Sq} is loopfull if and only if the condition (Def. 4) is satisfied.

(Def. 4) for every natural number n, $G_{Sq}(n)$ is loopfull.

Let us observe that every graph-yielding function which is empty is also loopfull and every graph-yielding function which is non empty and loopfull is also non loopless and there exists a graph sequence which is loopfull and there exists a graph-yielding finite sequence which is non empty and loopfull.

Let G_F be a loopfull, non empty, graph-yielding function and x be an element of $\operatorname{dom} G_F$. Note that $G_F(x)$ is loopfull.

Let G_{Sq} be a loopfull graph sequence and x be a natural number. Note that $G_{Sq}(x)$ is loopfull.

Let p be a loopfull, graph-yielding finite sequence and n be a natural number. Observe that $p \restriction n$ is loopfull and $p_{\lfloor n}$ is loopfull.

Let m be a natural number. One can check that $\operatorname{smid}(p, m, n)$ is loopfull and $\langle p(m), \ldots, p(n) \rangle$ is loopfull.

Let p, q be loopfull, graph-yielding finite sequences. Observe that $p \frown q$ is loopfull and $p \frown\!\frown q$ is loopfull.

Let G_1, G_2 be loopfull graphs. Note that $\langle G_1, G_2 \rangle$ is loopfull.

Let G_3 be a loopfull graph. Let us note that $\langle G_1, G_2, G_3 \rangle$ is loopfull.

2. Adding Loops to a Graph

Let G be a graph and V be a set.

A graph by adding a loop to each vertex of G in V is a supergraph of G defined by

(Def. 5) (i) the vertices of it = the vertices of G and there exists a set E and there exists a one-to-one function f such that E misses the edges of G and the edges of it = (the edges of G) $\cup E$ and $\operatorname{dom} f = E$ and $\operatorname{rng} f = V$ and the source of it = (the source of G)$+\cdot f$ and the target of it = (the target of G)$+\cdot f$, **if** $V \subseteq$ the vertices of G,

(ii) *it* $\approx G$, **otherwise**.

A graph by adding a loop to each vertex of G is a graph by adding a loop to each vertex of G in the vertices of G. Now we state the proposition:

(15) Let us consider a graph G_2, a set V, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then the vertices of G_1 = the vertices of G_2.

Let us consider a graph G_2, a set V, a graph G_1 by adding a loop to each vertex of G_2 in V, and objects e, v, w. Now we state the propositions:

(16) If $v \neq w$, then e joins v to w in G_1 iff e joins v to w in G_2.

(17) If $v \neq w$, then e joins v and w in G_1 iff e joins v and w in G_2. The theorem is a consequence of (16).

(18) Let us consider a graph G_2, a subset V of the vertices of G_2, a graph G_1 by adding a loop to each vertex of G_2 in V, and a vertex v of G_1. If $v \in V$, then v and v are adjacent.

(19) Let us consider a graph G_2, a set V, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then G_1.order() = G_2.order().

(20) Let us consider a graph G_2, a subset V of the vertices of G_2, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then G_1.size() = G_2.size()+ $\overline{\overline{V}}$.

(21) Let us consider graphs G_1, G_2. Then G_1 is a graph by adding a loop to each vertex of G_2 in $\emptyset$ if and only if $G_1 \approx G_2$. The theorem is a consequence of (15).

(22) Every graph is a graph by adding a loop to each vertex of G in $\emptyset$.

(23) Let us consider a graph G, subsets V_1, V_2 of the vertices of G, a graph G_1 by adding a loop to each vertex of G in V_1, and a graph G_2 by adding a loop to each vertex of G_1 in V_2. Suppose V_1 misses V_2. Then G_2 is a graph by adding a loop to each vertex of G in $V_1 \cup V_2$. The theorem is a consequence of (15).

(24) Let us consider a graph G_3, subsets V_1, V_2 of the vertices of G_3, and a graph G_1 by adding a loop to each vertex of G_3 in $V_1 \cup V_2$. Suppose V_1 misses V_2. Then there exists a graph G_2 by adding a loop to each vertex of G_3 in V_1 such that G_1 is a graph by adding a loop to each vertex of G_2 in V_2.

(25) Let us consider a loopless graph G_2, a subset V of the vertices of G_2, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then

(i) the edges of G_2 misses G_1.loops(), and

(ii) the edges of G_1 = (the edges of G_2) $\cup\, G_1$.loops().

(26) Let us consider a loopless graph G_1, a set V, a graph G_2 by adding a loop to each vertex of G_1 in V, and a subgraph G_3 of G_2 with loops removed. Then $G_1 \approx G_3$. The theorem is a consequence of (25).

(27) Let us consider graphs G_1, G_2, and a vertex v of G_2. Then G_1 is a graph by adding a loop to each vertex of G_2 in $\{v\}$ if and only if there exists an object e such that $e \notin$ the edges of G_2 and G_1 is a supergraph of G_2 extended by e between vertices v and v.

(28) Let us consider a graph G_2, a finite subset V of the vertices of G_2, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then there exists a non empty, graph-yielding finite sequence p such that

(i) $p(1) \approx G_2$, and

(ii) $p(\operatorname{len} p) = G_1$, and

(iii) $\operatorname{len} p = \overline{\overline{V}} + 1$, and

(iv) for every element n of $\operatorname{dom} p$ such that $n \leqslant \operatorname{len} p - 1$ there exists a vertex v of G_2 and there exists an object e such that $p(n+1)$ is a supergraph of $p(n)$ extended by e between vertices v and v and $v \in V$ and $e \notin$ the edges of $p(n)$.

Proof: Define $\mathcal{P}$[natural number] $\equiv$ for every graph G_2 for every finite subset V of the vertices of G_2 for every graph G_1 by adding a loop to each vertex of G_2 in V such that $\overline{\overline{V}} = \$_1$ there exists a non empty, graph-yielding finite sequence p such that $p(1) \approx G_2$ and $p(\operatorname{len} p) = G_1$ and $\operatorname{len} p = \overline{\overline{V}} + 1$.

For every element n of $\operatorname{dom} p$ such that $n \leqslant \operatorname{len} p - 1$ there exists a vertex v of G_2 and there exists an object e such that $p(n+1)$ is a supergraph of $p(n)$ extended by e between vertices v and v and $v \in V$ and $e \notin$ the edges of $p(n)$. $\mathcal{P}[0]$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. For every natural number k, $\mathcal{P}[k]$. $\square$

(29) Let us consider graphs G_3, G_4, sets V_1, V_2, a graph G_1 by adding a loop to each vertex of G_3 in V_1, a graph G_2 by adding a loop to each vertex of G_4 in V_2, and a partial graph mapping F_0 from G_3 to G_4. Suppose $V_1 \subseteq$ the vertices of G_3 and $V_2 \subseteq$ the vertices of G_4 and $F_{0\mathbb{V}} \restriction V_1$ is one-to-one and $\operatorname{dom}(F_{0\mathbb{V}} \restriction V_1) = V_1$ and $\operatorname{rng}(F_{0\mathbb{V}} \restriction V_1) = V_2$. Then there exists a partial graph mapping F from G_1 to G_2 such that

(i) $F_{\mathbb{V}} = F_{0\mathbb{V}}$, and

(ii) $F_{\mathbb{E}} \restriction \operatorname{dom}(F_{0\mathbb{E}}) = F_{0\mathbb{E}}$, and

(iii) if F_0 is not empty, then F is not empty, and

(iv) if F_0 is total, then F is total, and

(v) if F_0 is onto, then F is onto, and

(vi) if F_0 is one-to-one, then F is one-to-one, and

(vii) if F_0 is directed, then F is directed, and

(viii) if F_0 is weak subgraph embedding, then F is weak subgraph embedding, and

(ix) if F_0 is isomorphism, then F is isomorphism, and

(x) if F_0 is directed-isomorphism, then F is directed-isomorphism.

PROOF: Reconsider $f = F_{0\mathbb{V}}$ as a partial function from the vertices of G_1 to the vertices of G_2. Consider E_1 being a set, f_1 being a one-to-one function such that E_1 misses the edges of G_3 and the edges of $G_1 =$ (the edges of G_3) $\cup E_1$ and $\operatorname{dom} f_1 = E_1$ and $\operatorname{rng} f_1 = V_1$ and the source of $G_1 =$ (the source of G_3)$+\cdot f_1$ and the target of $G_1 =$ (the target of G_3)$+\cdot f_1$.

Consider E_2 being a set, f_2 being a one-to-one function such that E_2 misses the edges of G_4 and the edges of $G_2 =$ (the edges of G_4) $\cup E_2$ and $\operatorname{dom} f_2 = E_2$ and $\operatorname{rng} f_2 = V_2$ and the source of $G_2 =$ (the source of G_4)$+\cdot f_2$ and the target of $G_2 =$ (the target of G_4)$+\cdot f_2$. Set $h = {f_2}^{-1} \cdot (F_{0\mathbb{V}} \restriction V_1) \cdot f_1$. Set $g = F_{0\mathbb{E}} +\cdot h$. Reconsider $F = \langle f, g \rangle$ as a partial graph mapping from G_1 to G_2. If F_0 is total, then F is total. If F_0 is onto, then F is onto by [7, (6)]. If F_0 is one-to-one, then F is one-to-one. If F_0 is directed, then F is directed by [4, (70), (71)]. □

(30) Let us consider a graph G_3, a G_3-isomorphic graph G_4, and a graph G_1 by adding a loop to each vertex of G_3. Then every graph by adding a loop to each vertex of G_4 is G_1-isomorphic. The theorem is a consequence of (29).

(31) Let us consider a graph G_3, a G_3-directed-isomorphic graph G_4, and a graph G_1 by adding a loop to each vertex of G_3. Then every graph by adding a loop to each vertex of G_4 is G_1-directed-isomorphic. The theorem is a consequence of (29).

(32) Let us consider graphs G_3, G_4, a set V, a graph G_1 by adding a loop to each vertex of G_3 in V, and a graph G_2 by adding a loop to each vertex of G_4 in V. If $G_3 \approx G_4$, then G_2 is G_1-directed-isomorphic. The theorem is a consequence of (29).

(33) Let us consider a graph G_3, sets V, E, a graph G_4 given by reversing directions of the edges E of G_3, and a graph G_1 by adding a loop to each vertex of G_3 in V. Then every graph by adding a loop to each vertex of G_4 in V is G_1-isomorphic. The theorem is a consequence of (29).

(34) Let us consider a graph G_3, sets E, V, a graph G_4 given by reversing directions of the edges E of G_3, a graph G_1 by adding a loop to each vertex

of G_3 in V, and a graph G_2 given by reversing directions of the edges E of G_1. Suppose $E \subseteq$ the edges of G_3. Then G_2 is a graph by adding a loop to each vertex of G_4 in V. The theorem is a consequence of (15).

(35) Let us consider a graph G_3, a subset V_1 of the vertices of G_3, a non empty subset V_2 of the vertices of G_3, a subgraph G_4 of G_3 induced by V_2, and a graph G_1 by adding a loop to each vertex of G_3 in V_1. Then every subgraph of G_1 induced by V_2 is a graph by adding a loop to each vertex of G_4 in $V_1 \cap V_2$.

(36) Let us consider a graph G_2, a set V, a graph G_1 by adding a loop to each vertex of G_2 in V, a vertex v_1 of G_1, and a vertex v_2 of G_2. Suppose $v_1 \notin V$ and $v_1 = v_2$. Then

(i) v_1 is isolated iff v_2 is isolated, and

(ii) v_1 is endvertex iff v_2 is endvertex.

The theorem is a consequence of (17).

(37) Let us consider a graph G_2, a set V, a graph G_1 by adding a loop to each vertex of G_2 in V, and a path P of G_1. Then

(i) P is a path of G_2, or

(ii) there exist objects v, e such that e joins v and v in G_1 and $P = G_1.\mathrm{walkOf}(v, e, v)$.

The theorem is a consequence of (15).

(38) Let us consider a graph G_2, a set V, a graph G_1 by adding a loop to each vertex of G_2 in V, and a walk W of G_1. Suppose W.edges() misses (G_1.loops()) $\setminus$ (G_2.loops()). Then W is a walk of G_2. The theorem is a consequence of (15).

Let G be a graph. Observe that every graph by adding a loop to each vertex of G is loopfull.

Let V be a non empty subset of the vertices of G. Observe that every graph by adding a loop to each vertex of G in V is non loopless.

Now we state the proposition:

(39) Let us consider a graph G_2, a set V, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then G_1 is finite if and only if G_2 is finite. The theorem is a consequence of (15).

Let G be a finite graph and V be a set. Observe that every graph by adding a loop to each vertex of G in V is finite.

Let G be a non finite graph. Note that every graph by adding a loop to each vertex of G in V is non finite.

Now we state the proposition:

(40) Let us consider a graph G_2, a set V, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then G_1 is connected if and only if G_2 is connected. The theorem is a consequence of (15) and (37).

Let G be a connected graph and V be a set. Let us observe that every graph by adding a loop to each vertex of G in V is connected.

Let G be a non connected graph. Let us note that every graph by adding a loop to each vertex of G in V is non connected.

Now we state the proposition:

(41) Let us consider a graph G_2, a set V, and a graph G_1 by adding a loop to each vertex of G_2 in V. Then G_1 is chordal if and only if G_2 is chordal. The theorem is a consequence of (17) and (37).

Let G be a chordal graph and V be a set. Let us observe that every graph by adding a loop to each vertex of G in V is chordal.

Let G be a non edgeless graph. Let us note that every graph by adding a loop to each vertex of G in V is non edgeless.

Let G be a loopfull graph. Note that every graph by adding a loop to each vertex of G in V is loopfull.

Let G be a simple graph. Let us note that every graph by adding a loop to each vertex of G in V is non-multi.

Let G be a directed-simple graph. Note that every graph by adding a loop to each vertex of G in V is non-directed-multi.

Let us consider a graph G_2, a subset V of the vertices of G_2, a graph G_1 by adding a loop to each vertex of G_2 in V, a vertex v_1 of G_1, and a vertex v_2 of G_2. Now we state the propositions:

(42) Suppose $v_1 = v_2$ and $v_1 \in V$. Then there exists an object e such that

(i) e joins v_1 to v_1 in G_1, and

(ii) $e \notin$ the edges of G_2, and

(iii) v_1.edgesIn() $= v_2$.edgesIn() $\cup \{e\}$, and

(iv) v_1.edgesOut() $= v_2$.edgesOut() $\cup \{e\}$, and

(v) v_1.edgesInOut() $= v_2$.edgesInOut() $\cup \{e\}$.

(43) If $v_1 = v_2$ and $v_1 \in V$, then v_1.inDegree() $= v_2$.inDegree() $+ 1$ and v_1.outDegree() $= v_2$.outDegree() $+ 1$ and v_1.degree() $= v_2$.degree() $+ 2$. The theorem is a consequence of (42).

(44) Suppose $v_1 = v_2$ and $v_1 \notin V$. Then

(i) v_1.edgesIn() $= v_2$.edgesIn(), and

(ii) v_1.inDegree() $= v_2$.inDegree(), and

(iii) v_1.edgesOut() $= v_2$.edgesOut(), and

(iv) v_1.outDegree() $= v_2$.outDegree(), and

(v) v_1.edgesInOut() $= v_2$.edgesInOut(), and

(vi) v_1.degree() $= v_2$.degree().

3. Directed Graph Complement with Loops

Let G be a graph.

A directed graph complement of G with loops is a non-directed-multi graph defined by

(Def. 6) the vertices of *it* = the vertices of G and the edges of *it* misses the edges of G and for every vertices v, w of G, there exists an object e_1 such that e_1 joins v to w in G iff there exists no object e_2 such that e_2 joins v to w in *it*.

Now we state the proposition:

(45) Let us consider graphs G_1, G_2, G_3, and a directed graph complement G_4 of G_1 with loops. Suppose $G_1 \approx G_2$ and $G_3 \approx G_4$. Then G_3 is a directed graph complement of G_2 with loops.

Let G be a graph. Observe that there exists a directed graph complement of G with loops which is plain.

Now we state the propositions:

(46) Let us consider a graph G_1, a directed graph complement G_2 of G_1 with loops, and objects e_1, e_2, v, w. If e_1 joins v to w in G_1, then e_2 does not join v to w in G_2.

(47) Let us consider a graph G_1, and a subgraph G_2 of G_1 with directed-parallel edges removed. Then every directed graph complement of G_1 with loops is a directed graph complement of G_2 with loops. The theorem is a consequence of (46).

(48) Let us consider graphs G_1, G_2, a subgraph G_3 of G_1 with directed-parallel edges removed, a subgraph G_4 of G_2 with directed-parallel edges removed, a directed graph complement G_5 of G_1 with loops, and a directed graph complement G_6 of G_2 with loops. Suppose G_4 is G_3-directed-isomorphic. Then G_6 is G_5-directed-isomorphic. The theorem is a consequence of (47).

(49) Let us consider a graph G_1, a G_1-directed-isomorphic graph G_2, and a directed graph complement G_3 of G_1 with loops. Then every directed graph complement of G_2 with loops is G_3-directed-isomorphic. The theorem is a consequence of (48).

(50) Let us consider a graph G_1, and directed graph complements G_2, G_3 of G_1 with loops. Then G_3 is G_2-directed-isomorphic. The theorem is a consequence of (49).

(51) Let us consider a graph G_1, a graph G_2 given by reversing directions of the edges of G_1, and a directed graph complement G_3 of G_1 with loops. Then every graph given by reversing directions of the edges of G_3 is a directed graph complement of G_2 with loops. The theorem is a consequence of (46).

(52) Let us consider a graph G_1, a non empty subset V of the vertices of G_1, a subgraph G_2 of G_1 induced by V, and a directed graph complement G_3 of G_1 with loops. Then every subgraph of G_3 induced by V is a directed graph complement of G_2 with loops. The theorem is a consequence of (46).

(53) Let us consider a graph G_1, a proper subset V of the vertices of G_1, a subgraph G_2 of G_1 with vertices V removed, and a directed graph complement G_3 of G_1 with loops. Then every subgraph of G_3 with vertices V removed is a directed graph complement of G_2 with loops. The theorem is a consequence of (52).

(54) Let us consider a non-directed-multi graph G_1, and a directed graph complement G_2 of G_1 with loops. Then G_1 is a directed graph complement of G_2 with loops.

Let us consider a graph G_1 and a directed graph complement G_2 of G_1 with loops. Now we state the propositions:

(55) G_1.order() $=$ G_2.order().

(56) (i) G_1 is trivial iff G_2 is trivial, and

(ii) G_1 is loopfull iff G_2 is loopless, and

(iii) G_1 is loopless iff G_2 is loopfull.

The theorem is a consequence of (55), (1), and (46).

Let G be a trivial graph. One can verify that every directed graph complement of G with loops is trivial. Let G be a non trivial graph. One can check that every directed graph complement of G with loops is non trivial. Let G be a loopfull graph. Note that every directed graph complement of G with loops is loopless.

Let G be a non loopfull graph. Let us note that every directed graph complement of G with loops is non loopless. Let G be a loopless graph. Observe that every directed graph complement of G with loops is loopfull. Let G be a non loopless graph. Let us observe that every directed graph complement of G with loops is non loopfull.

Now we state the proposition:

(57) Let us consider a graph G_1, and a directed graph complement G_2 of G_1 with loops. Suppose the edges of $G_1 = G_1$.loops(). Then G_2 is complete.

Let G be an edgeless graph. One can verify that every directed graph complement of G with loops is complete. Let G be a non connected graph. One can check that every directed graph complement of G with loops is connected.

Now we state the propositions:

(58) Let us consider a graph G_1, a directed graph complement G_2 of G_1 with loops, a vertex v_1 of G_1, and a vertex v_2 of G_2. Suppose $v_1 = v_2$. Then

(i) if v_1 is isolated, then v_2 is not isolated, and

(ii) if v_1 is endvertex, then v_2 is not endvertex.

(59) Let us consider a graph G_1, a directed graph complement G_2 of G_1 with loops, and vertices v, w of G_1. Suppose there exists no object e such that e joins v and w in G_1. Then there exists an object e such that e joins v and w in G_2.

Proof: There exists no object e such that e joins v to w in G_1. Consider e being an object such that e joins v to w in G_2. □

Let us consider a graph G_1, a directed graph complement G_2 of G_1 with loops, a vertex v_1 of G_1, and a vertex v_2 of G_2. Now we state the propositions:

(60) Suppose $v_1 = v_2$. Then

(i) v_2.inNeighbors() = (the vertices of G_2) $\setminus$ (v_1.inNeighbors()), and

(ii) v_2.outNeighbors() = (the vertices of G_2) $\setminus$ (v_1.outNeighbors()).

(61) Suppose $v_1 = v_2$ and v_1 is isolated. Then

(i) v_2.inNeighbors() = the vertices of G_2, and

(ii) v_2.outNeighbors() = the vertices of G_2, and

(iii) v_2.allNeighbors() = the vertices of G_2.

The theorem is a consequence of (60).

4. Undirected Graph Complement with Loops

Let G be a graph.

An undirected graph complement of G with loops is a non-multi graph defined by

(Def. 7) the vertices of *it* = the vertices of G and the edges of *it* misses the edges of G and for every vertices v, w of G, there exists an object e_1 such that e_1 joins v and w in G iff there exists no object e_2 such that e_2 joins v and w in *it*.

Now we state the proposition:

(62) Let us consider graphs G_1, G_2, G_3, and an undirected graph complement G_4 of G_1 with loops. Suppose $G_1 \approx G_2$ and $G_3 \approx G_4$. Then G_3 is an undirected graph complement of G_2 with loops.

Let G be a graph. Note that there exists an undirected graph complement of G with loops which is plain.

Now we state the propositions:

(63) Let us consider a graph G_1, and a non-multi graph G_2. Then G_2 is an undirected graph complement of G_1 with loops if and only if the vertices of $G_2 =$ the vertices of G_1 and the edges of G_2 misses the edges of G_1 and for every vertices v_1, w_1 of G_1 and for every vertices v_2, w_2 of G_2 such that $v_1 = v_2$ and $w_1 = w_2$ holds v_1 and w_1 are adjacent iff v_2 and w_2 are not adjacent.

(64) Let us consider a graph G_1, an undirected graph complement G_2 of G_1 with loops, and objects e_1, e_2, v, w. If e_1 joins v and w in G_1, then e_2 does not join v and w in G_2.

(65) Let us consider a graph G_1, and a subgraph G_2 of G_1 with parallel edges removed. Then every undirected graph complement of G_1 with loops is an undirected graph complement of G_2 with loops. The theorem is a consequence of (64).

(66) Let us consider graphs G_1, G_2, a subgraph G_3 of G_1 with parallel edges removed, a subgraph G_4 of G_2 with parallel edges removed, an undirected graph complement G_5 of G_1 with loops, and an undirected graph complement G_6 of G_2 with loops. If G_4 is G_3-isomorphic, then G_6 is G_5-isomorphic. The theorem is a consequence of (65).

(67) Let us consider a graph G_1, a G_1-isomorphic graph G_2, and an undirected graph complement G_3 of G_1 with loops. Then every undirected graph complement of G_2 with loops is G_3-isomorphic. The theorem is a consequence of (66).

(68) Let us consider a graph G_1, and undirected graph complements G_2, G_3 of G_1 with loops. Then G_3 is G_2-isomorphic. The theorem is a consequence of (67).

(69) Let us consider a graph G_1, a non empty subset V of the vertices of G_1, a subgraph G_2 of G_1 induced by V, and an undirected graph complement G_3 of G_1 with loops. Then every subgraph of G_3 induced by V is an undirected graph complement of G_2 with loops. The theorem is a consequence of (64).

(70) Let us consider a graph G_1, a proper subset V of the vertices of G_1, a subgraph G_2 of G_1 with vertices V removed, and an undirected graph

complement G_3 of G_1 with loops. Then every subgraph of G_3 with vertices V removed is an undirected graph complement of G_2 with loops. The theorem is a consequence of (69).

(71) Let us consider a non-multi graph G_1, and an undirected graph complement G_2 of G_1 with loops. Then G_1 is an undirected graph complement of G_2 with loops.

Let us consider a graph G_1 and an undirected graph complement G_2 of G_1 with loops. Now we state the propositions:

(72) G_1.order() $= G_2$.order().

(73) (i) G_1 is trivial iff G_2 is trivial, and

(ii) G_1 is loopfull iff G_2 is loopless, and

(iii) G_1 is loopless iff G_2 is loopfull.

The theorem is a consequence of (72) and (64).

Let G be a trivial graph. Observe that every undirected graph complement of G with loops is trivial.

Let G be a non trivial graph. Let us observe that every undirected graph complement of G with loops is non trivial.

Let G be a loopfull graph. One can verify that every undirected graph complement of G with loops is loopless.

Let G be a non loopfull graph. One can check that every undirected graph complement of G with loops is non loopless.

Let G be a loopless graph. Note that every undirected graph complement of G with loops is loopfull.

Let G be a non loopless graph. Let us note that every undirected graph complement of G with loops is non loopfull.

Now we state the proposition:

(74) Let us consider a graph G_1, and an undirected graph complement G_2 of G_1 with loops. Suppose the edges of $G_1 = G_1$.loops(). Then G_2 is complete.

Let G be an edgeless graph. Observe that every undirected graph complement of G with loops is complete.

Now we state the proposition:

(75) Let us consider a complete graph G_1, and an undirected graph complement G_2 of G_1 with loops. Then the edges of $G_2 = G_2$.loops(). The theorem is a consequence of (64).

Let G be a complete, loopfull graph. Observe that every undirected graph complement of G with loops is edgeless.

Let G be a non connected graph. Note that every undirected graph complement of G with loops is connected.

Let us consider a graph G_1, an undirected graph complement G_2 of G_1 with loops, a vertex v_1 of G_1, and a vertex v_2 of G_2. Now we state the propositions:

(76) If $v_1 = v_2$, then if v_1 is isolated, then v_2 is not isolated and if v_1 is endvertex, then v_2 is not endvertex.

(77) If $v_1 = v_2$, then v_2.allNeighbors() = (the vertices of G_2)$\setminus$(v_1.allNeighbors ()).

(78) If $v_1 = v_2$ and v_1 is isolated, then v_2.allNeighbors() = the vertices of G_2. The theorem is a consequence of (77).

5. Directed Graph Complement without Loops

Let G be a graph.

A directed graph complement of G is a directed-simple graph defined by

(Def. 8) there exists a directed graph complement G' of G with loops such that *it* is a subgraph of G' with loops removed.

Now we state the proposition:

(79) Let us consider graphs G_1, G_2, G_3, and a directed graph complement G_4 of G_1. Suppose $G_1 \approx G_2$ and $G_3 \approx G_4$. Then G_3 is a directed graph complement of G_2. The theorem is a consequence of (45).

Let G be a graph. One can check that there exists a directed graph complement of G which is plain. Now we state the propositions:

(80) Let us consider a graph G_1, and a directed-simple graph G_2. Then G_2 is a directed graph complement of G_1 if and only if the vertices of G_2 = the vertices of G_1 and the edges of G_2 misses the edges of G_1 and for every vertices v, w of G_1 such that $v \neq w$ holds there exists an object e_1 such that e_1 joins v to w in G_1 iff there exists no object e_2 such that e_2 joins v to w in G_2. The theorem is a consequence of (46), (26), and (1).

(81) Let us consider a graph G_1, a directed graph complement G_2 of G_1, and objects e_1, e_2, v, w. If e_1 joins v to w in G_1, then e_2 does not join v to w in G_2. The theorem is a consequence of (80).

(82) Let us consider a graph G_1, and a directed-simple graph G_2 of G_1. Then every directed graph complement of G_1 is a directed graph complement of G_2. The theorem is a consequence of (80) and (81).

(83) Let us consider graphs G_1, G_2, a directed-simple graph G_3 of G_1, a directed-simple graph G_4 of G_2, a directed graph complement G_5 of G_1,

and a directed graph complement G_6 of G_2. Suppose G_4 is G_3-directed-isomorphic. Then G_6 is G_5-directed-isomorphic. The theorem is a consequence of (82) and (80).

(84) Let us consider a graph G_1, a G_1-directed-isomorphic graph G_2, and a directed graph complement G_3 of G_1. Then every directed graph complement of G_2 is G_3-directed-isomorphic. The theorem is a consequence of (83).

(85) Let us consider a graph G_1, and directed graph complements G_2, G_3 of G_1. Then G_3 is G_2-directed-isomorphic. The theorem is a consequence of (84).

(86) Let us consider a graph G_1, a graph G_2 given by reversing directions of the edges of G_1, and a directed graph complement G_3 of G_1. Then every graph given by reversing directions of the edges of G_3 is a directed graph complement of G_2. The theorem is a consequence of (80) and (81).

(87) Let us consider a graph G_1, a non empty subset V of the vertices of G_1, a subgraph G_2 of G_1 induced by V, and a directed graph complement G_3 of G_1. Then every subgraph of G_3 induced by V is a directed graph complement of G_2. The theorem is a consequence of (80) and (81).

(88) Let us consider a graph G_1, a proper subset V of the vertices of G_1, a subgraph G_2 of G_1 with vertices V removed, and a directed graph complement G_3 of G_1. Then every subgraph of G_3 with vertices V removed is a directed graph complement of G_2. The theorem is a consequence of (80) and (87).

(89) Let us consider a directed-simple graph G_1, and a directed graph complement G_2 of G_1. Then G_1 is a directed graph complement of G_2. The theorem is a consequence of (80).

Let us consider a graph G_1 and a directed graph complement G_2 of G_1. Now we state the propositions:

(90) $G_1.\mathrm{order}() = G_2.\mathrm{order}()$.

(91) G_1 is trivial if and only if G_2 is trivial. The theorem is a consequence of (90).

Let G be a trivial graph. One can verify that every directed graph complement of G is trivial. Let G be a non trivial graph. One can check that every directed graph complement of G is non trivial. Now we state the proposition:

(92) Let us consider a graph G_1, and a directed graph complement G_2 of G_1. Suppose the edges of $G_1 = G_1.\mathrm{loops}()$. Then G_2 is complete. The theorem is a consequence of (80).

Let G be an edgeless graph. One can check that every directed graph complement of G is complete. Let G be a trivial, edgeless graph. Let us observe that every directed graph complement of G is edgeless. Let G be a non connected graph. One can check that every directed graph complement of G is connected. Now we state the proposition:

(93) Let us consider a non trivial graph G_1, a directed graph complement G_2 of G_1, a vertex v_1 of G_1, and a vertex v_2 of G_2. If $v_1 = v_2$, then if v_1 is isolated, then v_2 is not isolated. The theorem is a consequence of (80).

Let us consider a graph G_1, a directed graph complement G_2 of G_1, a vertex v_1 of G_1, and a vertex v_2 of G_2. Now we state the propositions:

(94) If $v_1 = v_2$ and $3 \subseteq G_1$.order(), then if v_1 is endvertex, then v_2 is not endvertex.

PROOF: Consider u, w being vertices of G_1 such that $u \neq v_1$ and $w \neq v_1$ and $u \neq w$ and u and v_1 are adjacent and v_1 and w are not adjacent. There exists no object e such that e joins v_1 to w in G_1. Consider e_1 being an object such that e_1 joins v_1 to w in G_2. There exists no object e such that e joins w to v_1 in G_1. Consider e_2 being an object such that e_2 joins w to v_1 in G_2. Consider e' being an object such that v_2.edgesInOut() $= \{e'\}$ and e' does not join v_2 and v_2 in G_2. $\square$

(95) Suppose $v_1 = v_2$. Then

(i) v_2.inNeighbors() $=$ (the vertices of G_2) $\setminus$ (v_1.inNeighbors() $\cup \{v_2\}$), and

(ii) v_2.outNeighbors() $=$ (the vertices of G_2) $\setminus$ (v_1.outNeighbors() $\cup \{v_2\}$).

The theorem is a consequence of (60).

(96) Suppose $v_1 = v_2$ and v_1 is isolated. Then

(i) v_2.inNeighbors() $=$ (the vertices of G_2) $\setminus \{v_2\}$, and

(ii) v_2.outNeighbors() $=$ (the vertices of G_2) $\setminus \{v_2\}$, and

(iii) v_2.allNeighbors() $=$ (the vertices of G_2) $\setminus \{v_2\}$.

The theorem is a consequence of (95).

6. Undirected Graph Complement without Loops

Let G be a graph.

A graph complement of G is a simple graph defined by

(Def. 9) there exists an undirected graph complement G' of G with loops such that *it* is a subgraph of G' with loops removed.

Now we state the proposition:

(97) Let us consider graphs G_1, G_2, G_3, and a graph complement G_4 of G_1. Suppose $G_1 \approx G_2$ and $G_3 \approx G_4$. Then G_3 is a graph complement of G_2. The theorem is a consequence of (62).

Let G be a graph. Observe that there exists a graph complement of G which is plain. Let us consider a graph G_1 and a simple graph G_2. Now we state the propositions:

(98) G_2 is a graph complement of G_1 if and only if the vertices of G_2 = the vertices of G_1 and the edges of G_2 misses the edges of G_1 and for every vertices v, w of G_1 such that $v \neq w$ holds there exists an object e_1 such that e_1 joins v and w in G_1 iff there exists no object e_2 such that e_2 joins v and w in G_2. The theorem is a consequence of (64) and (26).

(99) G_2 is a graph complement of G_1 if and only if the vertices of G_2 = the vertices of G_1 and the edges of G_2 misses the edges of G_1 and for every vertices v_1, w_1 of G_1 and for every vertices v_2, w_2 of G_2 such that $v_1 = v_2$ and $w_1 = w_2$ and $v_1 \neq w_1$ holds v_1 and w_1 are adjacent iff v_2 and w_2 are not adjacent. The theorem is a consequence of (98).

(100) Let us consider a graph G_1, a graph complement G_2 of G_1, and objects e_1, e_2, v, w. If e_1 joins v and w in G_1, then e_2 does not join v and w in G_2. The theorem is a consequence of (98).

(101) Let us consider a graph G_1, and a simple graph G_2 of G_1. Then every graph complement of G_1 is a graph complement of G_2. The theorem is a consequence of (98) and (100).

(102) Let us consider graphs G_1, G_2, a simple graph G_3 of G_1, a simple graph G_4 of G_2, a graph complement G_5 of G_1, and a graph complement G_6 of G_2. If G_4 is G_3-isomorphic, then G_6 is G_5-isomorphic. The theorem is a consequence of (101) and (98).

(103) Let us consider a graph G_1, a G_1-isomorphic graph G_2, and a graph complement G_3 of G_1. Then every graph complement of G_2 is G_3-isomorphic. The theorem is a consequence of (102).

(104) Let us consider a graph G_1, and graph complements G_2, G_3 of G_1. Then G_3 is G_2-isomorphic. The theorem is a consequence of (103).

(105) Let us consider a graph G_1, an object v, a subset V of the vertices of G_1, a supergraph G_2 of G_1 extended by vertex v and edges between v and V of G_1, and a graph complement G_3 of G_1. Suppose $v \notin$ the vertices of G_1 and the edges of G_2 misses the edges of G_3. Then there exists a supergraph G_4 of G_3 extended by vertex v and edges between v and (the vertices of G_1) $\setminus V$ of G_3 such that G_4 is a graph complement of G_2. The theorem is a consequence of (98).

(106) Let us consider a graph G_1, an object v, a supergraph G_2 of G_1 extended

by v, and a graph complement G_3 of G_1. Suppose $v \notin$ the vertices of G_1. Then there exists a supergraph G_4 of G_3 extended by vertex v and edges between v and the vertices of G_3 such that G_4 is a graph complement of G_2. The theorem is a consequence of (98) and (105).

(107) Let us consider a graph G_1, an object v, a supergraph G_2 of G_1 extended by vertex v and edges between v and the vertices of G_1, a graph complement G_3 of G_1, and a supergraph G_4 of G_3 extended by v. Suppose $v \notin$ the vertices of G_1 and the edges of G_2 misses the edges of G_3. Then G_4 is a graph complement of G_2. The theorem is a consequence of (105) and (97).

(108) Let us consider a graph G_1, a non empty subset V of the vertices of G_1, a subgraph G_2 of G_1 induced by V, and a graph complement G_3 of G_1. Then every subgraph of G_3 induced by V is a graph complement of G_2. The theorem is a consequence of (98) and (100).

(109) Let us consider a graph G_1, a proper subset V of the vertices of G_1, a subgraph G_2 of G_1 with vertices V removed, and a graph complement G_3 of G_1. Then every subgraph of G_3 with vertices V removed is a graph complement of G_2. The theorem is a consequence of (98) and (108).

(110) Let us consider a simple graph G_1, and a graph complement G_2 of G_1. Then G_1 is a graph complement of G_2. The theorem is a consequence of (98).

Let us consider a graph G_1 and a graph complement G_2 of G_1. Now we state the propositions:

(111) G_1.order() $=$ G_2.order().

(112) G_1 is trivial if and only if G_2 is trivial. The theorem is a consequence of (111).

Let G be a trivial graph. Observe that every graph complement of G is trivial. Let G be a non trivial graph. Let us observe that every graph complement of G is non trivial. Now we state the proposition:

(113) Let us consider a graph G_1, and a graph complement G_2 of G_1. Then

(i) G_1 is complete iff G_2 is edgeless, and

(ii) the edges of $G_1 = G_1$.loops() iff G_2 is complete.

The theorem is a consequence of (99) and (98).

Let G be a complete graph. Observe that every graph complement of G is edgeless.

Let G be a non complete graph. Let us observe that every graph complement of G is non edgeless.

Let G be an edgeless graph. One can verify that every graph complement of G is complete.

Let G be a non connected graph. One can check that every graph complement of G is connected.

Now we state the propositions:

(114) Let us consider a non trivial graph G_1, a graph complement G_2 of G_1, a vertex v_1 of G_1, and a vertex v_2 of G_2. If $v_1 = v_2$, then if v_1 is isolated, then v_2 is not isolated. The theorem is a consequence of (98).

(115) Let us consider a graph G_1, a graph complement G_2 of G_1, a vertex v_1 of G_1, and a vertex v_2 of G_2. Suppose $v_1 = v_2$ and G_1.order() $= 2$. Then

(i) if v_1 is endvertex, then v_2 is isolated, and

(ii) if v_1 is isolated, then v_2 is endvertex.

The theorem is a consequence of (111), (98), and (100).

(116) Let us consider a simple graph G_1, a graph complement G_2 of G_1, a vertex v_1 of G_1, and a vertex v_2 of G_2. Suppose $v_1 = v_2$ and G_1.order() $= 2$. Then

(i) v_1 is endvertex iff v_2 is isolated, and

(ii) v_1 is isolated iff v_2 is endvertex.

The theorem is a consequence of (110), (111), and (115).

Let us consider a graph G_1, a graph complement G_2 of G_1, a vertex v_1 of G_1, and a vertex v_2 of G_2. Now we state the propositions:

(117) If $v_1 = v_2$ and $4 \subseteq G_1$.order(), then if v_1 is endvertex, then v_2 is not endvertex. The theorem is a consequence of (99).

(118) If $v_1 = v_2$, then v_2.allNeighbors() $=$ (the vertices of G_2)$\setminus$(v_1.allNeighbors() $\cup \{v_2\}$). The theorem is a consequence of (77).

(119) If $v_1 = v_2$ and v_1 is isolated, then v_2.allNeighbors() $=$ (the vertices of G_2) $\setminus \{v_2\}$. The theorem is a consequence of (118).

7. Self-complementary Graphs

Let G be a graph. We say that G is self-DLcomplementary if and only if

(Def. 10) every directed graph complement of G with loops is G-directed-isomorphic.

We say that G is self-Lcomplementary if and only if

(Def. 11) every undirected graph complement of G with loops is G-isomorphic.

We say that G is self-Dcomplementary if and only if

(Def. 12) every directed graph complement of G is G-directed-isomorphic.

We say that G is self-complementary if and only if

(Def. 13) every graph complement of G is G-isomorphic.

Let us consider a graph G. Now we state the propositions:

(120) G is self-DLcomplementary if and only if there exists a directed graph complement H of G with loops such that H is G-directed-isomorphic. The theorem is a consequence of (50).

(121) G is self-Lcomplementary if and only if there exists an undirected graph complement H of G with loops such that H is G-isomorphic. The theorem is a consequence of (68).

(122) G is self-Dcomplementary if and only if there exists a directed graph complement H of G such that H is G-directed-isomorphic. The theorem is a consequence of (85).

(123) G is self-complementary if and only if there exists a graph complement H of G such that H is G-isomorphic. The theorem is a consequence of (104).

Let us observe that every graph which is self-DLcomplementary is also non loopless, non loopfull, non-directed-multi, and connected and every graph which is self-Lcomplementary is also non loopless, non loopfull, non-multi, and connected and every graph which is self-Dcomplementary is also directed-simple and connected and every graph which is self-complementary is also simple and connected.

Every graph which is trivial and edgeless is also self-Dcomplementary and self-complementary and every graph which is self-Dcomplementary and self-complementary is also trivial and edgeless and every graph which is self-DLcomplementary is also non trivial, non self-Lcomplementary, non self-Dcomplementary, and non self-complementary and every graph which is self-Lcomplementary is also non trivial, non self-DLcomplementary, non self-Dcomplementary, and non self-complementary and there exists a graph which is self-Dcomplementary and self-complementary.

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