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Formal Development of Rough Inclusion Functions

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Summary. Rough sets, developed by Pawlak [15], are important tool to describe situation of incomplete or partially unknown information. In this article, continuing the formalization of rough sets [12], we give the formal characterization of three rough inclusion functions (RIFs). We start with the standard one, $\kappa^{\pounds}$, connected with Łukasiewicz [14], and extend this research for two additional RIFs: κ_1, and κ_2, following a paper by Gomolińska [4], [3]. We also define q-RIFs and weak q-RIFs [2]. The paper establishes a formal counterpart of [7] and makes a preliminary step towards rough mereology [16], [17] in Mizar [13].

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0. Introduction

In the paper, continuing our development of rough sets, we define three closely related rough inclusion functions (RIFs).

Until now, most of the Mizar formalization of rough sets [5], [8] was done by means of the notion of a generalized approximation space understood as a pair $\langle U, \rho \rangle$, where ρ is an indiscernibility relation defined on the universe U. This viewpoint, based on tolerances instead of equivalence relations, was studied by Skowron and Stepaniuk [18], then, in a general form, by Zhu [19], among many others, and the Mizar counterpart of it is included in [9] and [10].

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In the alternative approach, used by Gomolińska [3], approximation spaces are treated as triples of the form

$$\mathcal{A} = (U, I, \kappa),$$

where U is a non-empty set called the universe, $I : U \mapsto \wp U$ is an uncertainty mapping, and $\kappa : \wp U \times \wp U \mapsto [0,1]$ is a rough inclusion function. The formalization of uncertainty mappings was discussed in [12], and this article tries to define the missing part of the above definition, with future possibility of merging approaches via theory merging mechanism [6], avoiding duplication as much as we can [11].

We start with some preliminaries, which cover gaps in the existing state of the Mizar Mathematical Library. Section 2 deals with the standard rough inclusion function, which appears in some form in the research by Jan Łukasiewicz [14], obviously without any reference for rough sets. This pretty general Mizar functor $\kappa^{\pounds}$ is defined as follows:

$$\kappa^{\pounds}(X,Y) = \begin{cases} \frac{|X \cap Y|}{|X|}, & \text{if } X \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

For a given universe U, rough inclusion functions (RIFs for short) are the mappings κ from $\wp U \times \wp U$ into unit interval which satisfy two properties:

$$\mathrm{rif}_1(\kappa) \Leftrightarrow \forall_{X,Y \subseteq U}\ (\kappa(X,Y) = 1 \Leftrightarrow X \subseteq Y)$$

$$\mathrm{rif}_2(\kappa) \Leftrightarrow \forall_{X,Y,Z \subseteq U}\ (Y \subseteq Z \Rightarrow \kappa(X,Y) \leqslant \kappa(X,Z))$$

This is discussed in Sect. 3; corresponding Mizar modes `RIF` and `preRIF` are also introduced.

Besides $\kappa^{\pounds}$, there are two relatively well-known RIFs:

$$\kappa_1(X,Y) = \begin{cases} \frac{|Y|}{|X \cup Y|}, & \text{if } X \cup Y \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

$$\kappa_2(X,Y) = \frac{|(U - X) \cup Y|}{|U|}.$$

Section 4 contains their definitions, both of the form of Mizar functors, and as set-theoretic functions.

It should be mentioned that in this Mizar translation (and also in the source code), predicative form of the properties of RIFs, as, for example, $\mathrm{rif}_1(\kappa)$ is replaced by the phrase "κ satisfies (RIF_1)" (and for others, respectively). In Sect. 5 we formulate some additional characteristic properties of rough inclusions; in Sect. 6 we show that, under the assumption that rif_1 holds, rif_2 can be replaced by rif_2^*. We introduce also some weakened versions of rough inclusions: quasi-RIF and weak quasi-RIF.

All three considered RIFs ($\kappa^{\pounds}, \kappa_1, \kappa_2$) are distinct. Gomolińska takes $U = \{0, 1, 2, \ldots, 9\}$, $X = \{0, \ldots, 4\}$, $Y = \{2, \ldots, 6\}$. Then $\kappa^{\pounds}(X, Y) = 3/5$, $\kappa_1(X, Y) = 5/7$, and $\kappa_2(X, Y) = 4/5$. In Sect. 9, we constructed an example, which in Mizar is the functor `ExampleRIFSpace`, claiming that $U = \{1, 2, 3, 4, 5\}$, $X = \{1, 2\}$, $Y = \{2, 3, 4\}$ with $\kappa^{\pounds}(X, Y) = 1/2$, $\kappa_1(X, Y) = 3/4$, and $\kappa_2(X, Y) = 4/5$. Obviously, the indiscernibility relation does not matter, and so we took the identity as the simplest. The proofs, based on our specific example, are significantly shorter than those proposed by Gomolińska.

In the final section, we formalized two theorems from another Gomolińska's paper [1], which was already translated into Mizar [12], but without the notion of RIF; now we can fill this gap.

1. Preliminaries

Now we state the propositions:

(1) Let us consider real numbers a, b, c. If $b > 0$ and $a \leqslant b$ and $c \geqslant 0$, then $\frac{a}{b} \leqslant \frac{a+c}{b+c}$.

Observe that there exists an approximation space which is strict and finite.

Let R be a finite 1-sorted structure. One can check that every subset of R is finite.

From now on R denotes a 1-sorted structure and X, Y denote subsets of R.

(2) $X \subseteq Y$ if and only if $X^{\mathrm{c}} \cup Y = \Omega_R$.

PROOF: If $X \subseteq Y$, then $X^{\mathrm{c}} \cup Y = \Omega_R$. □

From now on R denotes a finite 1-sorted structure and X, Y denote subsets of R. Now we state the propositions:

(3) $\overline{\overline{X \cup Y}} = \overline{\overline{Y}}$ if and only if $X \subseteq Y$.

(4) If $\overline{\overline{X^{\mathrm{c}} \cup Y}} = \overline{\overline{\Omega_R}}$, then $X^{\mathrm{c}} \cup Y = \Omega_R$.

Let R be a non empty 1-sorted structure and X be a subset of R. Note that $\Omega_R \cup X$ reduces to Ω_R and $\Omega_R \cap X$ reduces to X.

2. Standard Rough Inclusion Function

From now on R denotes a finite approximation space and X, Y, Z, W denote subsets of R.

Let R be a finite approximation space and X, Y be subsets of R. The functor $\kappa^{\pounds}(X, Y)$ yielding an element of $[0, 1]$ is defined by the term

(Def. 1) $\begin{cases} \frac{\overline{\overline{X \cap Y}}}{\overline{\overline{X}}}, & \textbf{if } X \neq \emptyset, \\ 1, & \textbf{otherwise}. \end{cases}$

Now we state the propositions:

(5) $\kappa^{\pounds}(\emptyset_R, X) = 1$.

(6) $\kappa^{\pounds}(X, Y) = 1$ if and only if $X \subseteq Y$.

(7) If $Y \subseteq Z$, then $\kappa^{\pounds}(X, Y) \leqslant \kappa^{\pounds}(X, Z)$.

(8) If $Z \subseteq Y \subseteq X$, then $\kappa^{\pounds}(X, Z) \leqslant \kappa^{\pounds}(Y, Z)$.

(9) $\kappa^{\pounds}(X, Y \cup Z) \leqslant \kappa^{\pounds}(X, Y) + \kappa^{\pounds}(X, Z)$.

(10) If $X \neq \emptyset$ and Y misses Z, then $\kappa^{\pounds}(X, Y \cup Z) = \kappa^{\pounds}(X, Y) + \kappa^{\pounds}(X, Z)$.

3. Rough Inclusion Functions

Let R be a 1-sorted structure.

A pre-rough inclusion function of R is a function from $2^{(\text{the carrier of } R)} \times 2^{(\text{the carrier of } R)}$ into $[0, 1]$.

A preRIF of R is a pre-rough inclusion function of R.

The scheme *BinOpEq* deals with a non empty 1-sorted structure $\mathcal{R}$ and a binary functor $\mathcal{F}$ yielding an element of $[0, 1]$ and states that

(Sch. 1) For every preRIFs f_1, f_2 of $\mathcal{R}$ such that for every subsets x, y of $\mathcal{R}$, $f_1(x, y) = \mathcal{F}(x, y)$ and for every subsets x, y of $\mathcal{R}$, $f_2(x, y) = \mathcal{F}(x, y)$ holds $f_1 = f_2$.

Let R be a finite approximation space. The functor $\kappa^{\pounds}(R)$ yielding a preRIF of R is defined by

(Def. 2) for every subsets x, y of R, $it(x, y) = \kappa^{\pounds}(x, y)$.

4. Defining Two New RIFs

Let R be a finite approximation space and X, Y be subsets of R. The functor $\kappa_1(X, Y)$ yielding an element of $[0, 1]$ is defined by the term

(Def. 3) $\begin{cases} \frac{\overline{\overline{Y}}}{\overline{\overline{X \cup Y}}}, & \textbf{if } X \cup Y \neq \emptyset, \\ 1, & \textbf{otherwise}. \end{cases}$

The functor $\kappa_2(X, Y)$ yielding an element of $[0, 1]$ is defined by the term

(Def. 4) $\frac{\overline{\overline{X^c \cup Y}}}{\overline{\overline{\Omega_R}}}$.

The functor $\kappa_1(R)$ yielding a preRIF of R is defined by

(Def. 5) for every subsets x, y of R, $it(x, y) = \kappa_1(x, y)$.

The functor $\kappa_2(R)$ yielding a preRIF of R is defined by

(Def. 6) for every subsets x, y of R, $it(x, y) = \kappa_2(x, y)$.

Now we state the propositions:

(11) $\kappa_1(X, Y) = 1$ if and only if $X \subseteq Y$. The theorem is a consequence of (3).

(12) $\kappa_2(X, Y) = 1$ if and only if $X \subseteq Y$. The theorem is a consequence of (2) and (4).

(13) If $\kappa_1(X, Y) = 0$, then $Y = \emptyset$.

(14) If $X \neq \emptyset$, then $\kappa_1(X, Y) = 0$ iff $Y = \emptyset$. The theorem is a consequence of (13).

(15) $\kappa_2(X, Y) = 0$ if and only if $X = \Omega_R$ and $Y = \emptyset$.

(16) If $Y \subseteq Z$, then $\kappa_1(X, Y) \leqslant \kappa_1(X, Z)$. The theorem is a consequence of (1).

(17) If $Y \subseteq Z$, then $\kappa_2(X, Y) \leqslant \kappa_2(X, Z)$.

(18) $\kappa_1(\emptyset_R, X) = 1$. The theorem is a consequence of (11).

(19) $\kappa_2(\emptyset_R, X) = 1$. The theorem is a consequence of (12).

5. Characteristic Properties of Rough Inclusions

Let R be a non empty relational structure and κ be a preRIF of R. We say that κ satisfies (RIF_1) if and only if

(Def. 7) for every subsets X, Y of R, $\kappa(X, Y) = 1$ iff $X \subseteq Y$.

We say that κ satisfies (RIF_2) if and only if

(Def. 8) for every subsets X, Y, Z of R such that $Y \subseteq Z$ holds $\kappa(X, Y) \leqslant \kappa(X, Z)$.

We say that κ satisfies (RIF_3) if and only if

(Def. 9) for every subset X of R such that $X \neq \emptyset$ holds $\kappa(X, \emptyset_R) = 0$.

We say that κ satisfies (RIF_4) if and only if

(Def. 10) for every subsets X, Y of R such that $\kappa(X, Y) = 0$ holds X misses Y.

We say that κ satisfies (RIF_0) if and only if

(Def. 11) for every subsets X, Y of R such that $X \subseteq Y$ holds $\kappa(X, Y) = 1$.

We say that κ satisfies (RIF_0^{-1}) if and only if

(Def. 12) for every subsets X, Y of R such that $\kappa(X, Y) = 1$ holds $X \subseteq Y$.

We say that κ satisfies (RIF_2^*) if and only if

(Def. 13) for every subsets X, Y, Z of R such that $\kappa(Y, Z) = 1$ holds $\kappa(X, Y) \leqslant \kappa(X, Z)$.

Observe that every preRIF of R which satisfies (RIF_1) satisfies also (RIF_0) and (RIF_0^{-1}) and every preRIF of R which satisfies (RIF_0) and (RIF_0^{-1}) satisfies also (RIF_1).

Let R be a finite approximation space. One can check that $\kappa^{\pounds}(R)$ satisfies (RIF_1) and $\kappa^{\pounds}(R)$ satisfies (RIF_2) and $\kappa_1(R)$ satisfies (RIF_1) and $\kappa_1(R)$ satisfies (RIF_2) and $\kappa_2(R)$ satisfies (RIF_1) and $\kappa_2(R)$ satisfies (RIF_2).

Let us consider R. Note that there exists a preRIF of R which satisfies (RIF_1) and (RIF_2).

6. On the Connections between Postulates

Now we state the proposition:

(20) Let us consider preRIF κ of R satisfying (RIF_1). Then κ satisfies (RIF_2) if and only if κ satisfies (RIF_2^*).

Let us consider R. Let us observe that every preRIF of R satisfying (RIF_1) which satisfies (RIF_2) satisfies also (RIF_2^*) and every preRIF of R satisfying (RIF_1) which satisfies (RIF_2^*) satisfies also (RIF_2) and $\kappa^{\pounds}(R)$ satisfies (RIF_0) and (RIF_2^*) and there exists a pre-rough inclusion function of R which satisfies (RIF_0), (RIF_1), (RIF_2), and (RIF_2^*).

A rough inclusion function of R is pre-rough inclusion function of R satisfying (RIF_1) and (RIF_2).

A quasi-rough inclusion function of R is preRIF of R satisfying (RIF_0) and (RIF_2^*).

A weak quasi-rough inclusion function of R is preRIF of R satisfying (RIF_0) and (RIF_2).

A RIF of R is a rough inclusion function of R.

A q-RIF of R is a quasi-rough inclusion function of R.

A weak q-RIF of R is a weak quasi-rough inclusion function of R.

7. Formalization of Proposition 2 [3]

Now we state the propositions:

(21) If $X \neq \emptyset$ and $Z \cup W = \Omega_R$ and Z misses W, then $\kappa^{\pounds}(X, Z) + \kappa^{\pounds}(X, W) = 1$.

(22) If $\kappa^{\pounds}(X, Y) = 0$, then X misses Y.

(23) If $X \neq \emptyset$, then $\kappa^{\pounds}(X, Y) = 0$ iff X misses Y. The theorem is a consequence of (22).

(24) If $X \neq \emptyset$, then $\kappa^{\pounds}(X, \emptyset_R) = 0$.

Now we state the propositions:

(25) If $X \neq \emptyset$ and X misses Y, then $\kappa^{£}(X, Z \setminus Y) = \kappa^{£}(X, Z \cup Y) = \kappa^{£}(X, Z)$. The theorem is a consequence of (23), (10), (7), and (9).

(26) If Z misses W, then $\kappa^{£}(Y \cup Z, W) \leqslant \kappa^{£}(Y, W) \leqslant \kappa^{£}(Y \setminus Z, W)$.

(27) If Z misses Y and $Z \subseteq W$, then $\kappa^{£}(Y \setminus Z, W) \leqslant \kappa^{£}(Y, W) \leqslant \kappa^{£}(Y \cup Z, W)$.

8. Formalization of Proposition 4 [3]

Let us consider R. Let X be a non empty subset of R. Let us note that $\kappa^{£}(X, \emptyset_R)$ is empty.

Now we state the propositions:

(28) If $\kappa_1(X, Y) = 0$, then X misses Y. The theorem is a consequence of (14).

(29) If $\kappa_2(X, Y) = 0$, then X misses Y. The theorem is a consequence of (15).

Let us consider R. Observe that $\kappa^{£}(R)$ satisfies (RIF_4) and $\kappa_1(R)$ satisfies (RIF_4) and $\kappa_2(R)$ satisfies (RIF_4).

(30) $\kappa^{£}(X, Y) \leqslant \kappa_1(X, Y) \leqslant \kappa_2(X, Y)$. The theorem is a consequence of (1), (18), and (19).

(31) $\kappa_1(X, Y) = \kappa^{£}(X \cup Y, Y)$. The theorem is a consequence of (6) and (11).

(32) $\kappa_2(X, Y) = \kappa^{£}(\Omega_R, X^{c} \cup Y) = \kappa^{£}(\Omega_R, X^{c}) + \kappa^{£}(\Omega_R, X \cap Y)$.

(33) $\kappa^{£}(X, Y) = \kappa^{£}(X, X \cap Y) = \kappa_1(X, X \cap Y) = \kappa_1(X \setminus Y, X \cap Y)$.

(34) If $X \cup Y = \Omega_R$, then $\kappa_1(X, Y) = \kappa_2(X, Y)$. The theorem is a consequence of (2).

(35) If $X \neq \emptyset$, then $1 - \kappa^{£}(X, Y) = \kappa^{£}(X, Y^{c})$. The theorem is a consequence of (10) and (6).

9. Concrete Example

Let X be a set. The functor DiscreteApproxSpace(X) yielding a strict relational structure is defined by the term

(Def. 14) $\langle X, \mathrm{id}_X \rangle$.

Let us note that DiscreteApproxSpace(X) has equivalence relation.

Let X be a non empty set. Observe that DiscreteApproxSpace(X) is non empty.

Let X be a finite set. Let us observe that DiscreteApproxSpace(X) is finite.

The functor ExampleRIFSpace yielding a strict, finite approximation space is defined by the term

(Def. 15) DiscreteApproxSpace($\{1,2,3,4,5\}$).

Now we state the propositions:

(36) Let us consider subsets X, Y of ExampleRIFSpace. Suppose $X=\{1,2\}$ and $Y=\{2,3,4\}$. Then $\kappa^{\pounds}(X,Y)\neq\kappa^{\pounds}(Y,X)$.

(37) Let us consider subsets X, Y, U of ExampleRIFSpace. Suppose $X=\{1,2\}$ and $Y=\{1,2,3\}$ and $U=\{2,4,5\}$. Then $\kappa^{\pounds}(X,U)\not\leqslant\kappa^{\pounds}(Y,U)$.

(38) Let us consider subsets X, Y of ExampleRIFSpace. Suppose $X=\{1,2\}$ and $Y=\{2,3,4\}$. Then $\kappa^{\pounds}(X,Y)$, $\kappa_1(X,Y)$, $\kappa_2(X,Y)$ are mutually different.

10. Continuing Formalization of Theorem 4.1 [1]

Let us consider a finite approximation space R, an element u of R, and subsets x, y of R. Now we state the propositions:

(39) If $u\in(f_1(R))(x)$ and $(I_R)(u)=y$, then $\kappa^{\pounds}(y,x)>0$. The theorem is a consequence of (22).

(40) If $u\in(\mathrm{Flip}\,f_1(R))(x)$ and $(I_R)(u)=y$, then $\kappa^{\pounds}(y,x)=1$. The theorem is a consequence of (6).

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