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ARITHMOLOGY AND MONADOLOGY OF NIKOLAI BUGAEV

Abstract

Nikolai Bugaev was a mathematician keenly interested in philosophy. He stressed the role of discontinuity in his mathematical research that he called arithmology. He also emphasized the importance of discontinuity in nature which he embodied in his version of monadology. The article discusses the viability of his philosophical investigations.

Keywords: arithmology, monadology, continuity, discontinuity, Leibniz, Solovyov

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Nikolai Vasil'evich Bugaev (1837–1903) was an important figure in the intellectual life of the end of 19th century Moscow. He was a co-founder of the important Moscow Mathematical Society (established in 1864, officially, in 1867) of which he was a vice-president and then a president. He was a professor at the Moscow University in which he was also for several years a dean of the physics and mathematics division. A creative mathematician, he was also keenly interested in philosophy in which he contributed a version of monadology, an extension and modification of the monadology of Leibniz.

Arithmology

Bugaev's interests in mathematics were influenced by his philosophical views. He thought that the reigning mathematical paradigm relied on the differential and integral calculus that did marvels to the development of science in the 18th and 19th centuries. Such wide and successful application of its analysis in physics led to the emergence of the analytical worldview in which natural processes were viewed as continuous and they have been modeled by continuous functions of calculus (MiN 704).[1] However, all natural processes are not continuous and Bugaev urged the mathematical community to investigate also discontinuous functions, an area of mathematics for which he proposed the name of arithmology (arithmologia) (699), a rather unfortunate choice of terminology since it invokes the name of numerology and so does the name arithmologia,[2] the name already used by, for instance, Athanasius Kircher in the 17th century for numerological purposes.[3]

The division between continuous and discontinuous functions is rather insufficient for what Bugaev really meant. Discontinuous functions can very well be in the purview of traditional calculus: consider the concepts of left and right derivatives and integrals being sums of integrals for continuous intervals of functions, the integrals already investigated by Cauchy. E.g., function $f(x) = 1/x$ is discontinuous at $x = 0$ and yet it can perfectly well be analyzed by calculus.

[1] The following references will be used:

BM – Н. В. Бугаев, Основныя начала эволюционной монадологии, *Вопросы философии и психологии* 4 (1893), no. 2 (17), March, pp. 26-44.

LM – Gottfried W. Leibniz, *Monadology* 1714.

MiN – Н. В. Бугаев, Математика и научно-философское миросозерцание, *Вопросы философии и психологии* 9 (1898), no. 5 (45), pp. 697-717.

SV – Н. В. Бугаев, О свободе воли, in: Н.И Грот (ed.), *О свободе воли: Опыты постановки и решения вопроса: рефераты и статьи членов Психологическаго общества*, Москва: Типография А. Гатцука 1889, pp. 195-218.

[2] A transliteration of the Russian name аритмология used by Bugaev. Bugaev may not have been opposed to the numerical aspect of mathematics. At one point, he briefly mentioned that truths of analysis are general, truths of arithmology are individual, and they draw the attention of some thinkers by their mysteriousness leading them to some mystical speculations on numbers (MiN 701).

[3] On the other hand, it was the name Ampère used in his classification of sciences: mathematics was divided into arithmology and geometry, the former into 1. elementary arithmology which encompassed arithmography and mathematical analysis, and 2. megenthology which was subdivided into theory of functions and probability theory, André-Marie Ampère, *Essai sur la philosophie des sciences*, Paris: Bachelier, vol. 1, 1834, pp. 40-41; cf. James R. Hofmann, André-Marie Ampère, Cambridge: Cambridge University Press 1996, pp. 363-364.

Even a function with an infinite number of discontinuities, e.g., the function $f(x) = 0$ if $\lfloor x \rfloor$ is an odd number and 1 if it is even; that is, it is 0 if x falls between an odd integer and the following even integer, and 1 if x is between an even integer and the next odd integer. This, to be sure, can be carried to the extreme by allowing $f(x) = 0$ if x is an irrational number and 1 if x is rational, thereby obtaining the Dirichlet function, an example of a function which is everywhere discontinuous. However, this is not the type of discontinuity Bugaev meant. He provided a simple example of a function $\rho(n)$ = the number of divisors of n, which is a nonanalytical function, since, e.g., for numbers between 2 and 3 the function is meaningless.[4] More generally, for a function $y = f(x)$, Bugaev meant the discontinuity of the independent variable x rather than the dependent variable y. However, this must be, as it were, a discontinuity all the way. Thus, function $f(x) = 1/x$ would not qualify although it is not defined for $x = 0$, or the trigonometrical function $\tan(x)$, undefined for $x = \pm i\pi/2$ where i is an odd integer, that is, with an infinite number of discontinuities on the x-axis, and yet it is a perfectly analytical function. Alekseev gave an example of a function that measures thermal conductivity in a body. For a uniform body, the function is continuous and thus analytical. For a non-uniform body, there will be jumps and thus arithmology should be applied.[5] However, the example was ill-chosen. The function for a non-uniform body will have discontinuities, but, still, it can very well be analyzed by traditional calculus.

To see an arithmological example, Bugaev's function $\rho(n)$ points the way: the function is defined only for integers, that is, for each value of the independent variable n there is a discontinuity on the x-axis – and consequently on the y-axis. This becomes even more clear if we consider his definitions of algebraic integral and derivative.

For an arbitrary function $f(n)$, the arithmetic integral of $f(n)$ over natural numbers is simply the sum $F(n) = \sum_{1 \leq k \leq n} f(k)$. For the sum $F(n) = \sum_{k \mid n} f(k)$ over all divisors k of n, the function $f(n)$ is the arithmetic derivative of function $F(n)$.[6] These were the fundamental definitions of Bugaev's arithmology, which today we would consider an area of the discrete calculus. As a mathematician, Bugaev

[4] Н. В. Бугаев, Введение в теорию чисел (вступительная лекция), *Математический сборник* 25 (1905), no. 2, p. 339.

[5] В[иссарион] Г. Алексеев, *Н.В. Бугаев и проблемы идеализма Московской математической школы*, Юрьев: Типография К. Маттисена 1905, pp. 30-31.

[6] Н. В. Бугаев, Учение о числовых производных, *Математический сборник* 5 (1870), no. 1, pp. 1-2.

was particularly active in his arithmology and published many articles on the subject. For example, he investigated various arithmetic identities using properties of arithmetic derivatives and integrals. He was working on the representation of arithmetic functions as infinite series; for example, he showed that any function $F(n)$ for natural numbers n and $F(0) = 0$, the function can be represented by an series $F(n) = \sum_{1 \leq k \leq n} f(k)E(n/k)$, where $E(x)$ is an integer part of x – i.e., $E(x) = \lfloor x \rfloor$ – and an arithmetic derivative $f(n) = (F(n) - F(n-1))'$.[7] He was working on infinite products; for example, he found that for the product $T(n)$ of primes < n, $T(n) = \prod_v(\prod_u(n^{1/u}/v))^{q(u)q(v)}$, for $q(u)$ equal to 0, –1 or 1.[8] He wanted to create methods for number theory which would be as general as the methods of calculus. He was also working with more conventional calculus, for instance, on convergence of infinite series, finding algebraic integrals for differential equations, and on successive approximation methods.

Bugaev's investigations on discrete functions have some mathematical significance; however, he tried to do too much about its philosophical significance, and confusingly at that. First, the definition: arithmology is defined as the mathematical analysis of discontinuous functions (MiN 699) which we should understand as the analysis of functions $f(n)$ where n is an integer (more generally, it could be an analysis of functions $f(x)$ for everywhere discontinuous values of x) – the discrete calculus. On the other hand, it is defined as the number theory (700), but the two domains do not quite coincide. Bugaev's pupil, Nekrasov, defined arithmology as the theory of discontinuous functions and of numbers, emphasizing that this is a theory of discontinuous functions and not only a theory of numbers.[9]

In Bugaev's view, the analytical worldview leads in biology to a conviction that the end result of biological phenomena is a result of infinitely small changes. In sociology it leads to a view that the social growth takes place through slow and continuous progress of all elements of society. Also, evolutionism replaces revolutionary theory in history because of the conviction that social improvement takes place not by social jumps, but through steady improvements (MiN 706). In the analytical worldview, a conviction arose that in nature causes are

[7] Бугаев, Учение, pp. 12-13; А[лександр] П. Минин, О трудах Н.В. Бугаева по теории чисел, *Математический сборник* 25 (1905), p. 302.

[8] Н. В. Бугаев, Учение о числовых производных. [part] III, *Математический сборник* 6 (1873), no. 3, p. 250; Минин, *op. cit.*, p. 310.

[9] П. А. Некрасов, Московская философско-математическая школа и ея основатели, *Математический сборник* 25 (1904), no. 1, pp. 24, 246.

important in events, not the goal, not the moral values of good and evil, not the esthetic value of beauty, not freedom, not justice – all of them being only illusions; many espouse fatalism and complete determinism (707). However, continuous functions are not always applicable in chemistry; each body is an individual entity because of its makeup. Arithmology should be used instead (709). Crystals cannot be explained by continuity. Also, only some sounds are esthetically pleasing. The cellular makeup of bodies does not fit the analytical worldview. Not all social phenomena can be explained by continuity. Discontinuity appears when there are self-standing individuals and when goal-orientation is taken into account and when there are esthetic and ethical issues. So, the analytical worldview is limited (710). The arithmological worldview acknowledges that good, evil, and beauty are not illusory and belong to the roots of things. Fatalism disappears. This worldview is added to the analytical view, it does not replace it. A mathematical worldview changes the view of progress (711).

Arithmology also shows, in Bugaev's view, that the use of mathematics may not be tantamount to remaining in the deterministic context and he pointed to inverses of at least some arithomological functions (MiN 712-713); for example, for the aforementioned function $y = E(x)$, an inverse $x = E^{-1}(y)$ would not provide a unique value x for an integer y, but an infinity of numbers from an interval $[y, y+1)$.[10] The phenomenon is not, of course, unique to discrete functions. Consider the inverse of $y = x^2$ which would have two values, and thus only positive values are retained in the definition of its inverse, $\sqrt{y}$. Consider $y = \sin(x)$ and its inverse $\arcsin(y)$ whose domain by definition is limited to the interval $[-1, 1]$, and thus the range becomes only the interval $[-\pi/2, \pi/2]$ since there would be an infinity of values for the same y.

In this call to the use of arithmology, Bugaev unjustifiably identified arithmology with discontinuity. Consider the social progress. Whether it takes place by small gradual changes or by abrupt changes, the analytical worldview can embrace both. If social progress could be expressed by some function $y = f(t)$, where y is the level of progress, t would be time, continuous time, that is, possible jumps in the level of progress would pertain to y, and calculus could handle such functions. The example could be saved when sampling is intro-

[10] Cf. В[ладислав] А. Шапошников, Философские взгляды Н.В. Бугаева и русская культура конца XIX – начала XX вв., *Историко-математические исследования*, Вторая серия, 7 (42) (2002), p. 76.

duced – which would be necessary if we wanted the measurement to be made and processed digitally by the computer: so, a measurement of the progress level would be taken once, say, a month, or a year, or according to some other sampling rate. In this case, discontinuity would be introduced on the x-axis (or rather the time t-axis), which means very little, since Bugaev is interested in discontinuities in the level of progress, not in the frequency of taking samples of this level. Thus, in all this, Bugaev's arithmological mathematics has very little to do with what he called the arithmological worldview. Maybe the name "the *natura facit saltus* worldview" would fit his intentions better considering the presence of Leibniz in Bugaev's philosophy and the fact that for Leibniz, *natura not fact saltus* (*New essays* 4.16).

It is also unclear why rejection of discontinuity should lead to rejection of a goal, goodness, beauty, or freedom. Could not continuous processes be goal directed? or aiming at good? or at the beautiful? Why could freedom not be expressed by a continuous trajectory if the course of this trajectory were not predetermined and depended only one a person's free will? By Bugaev's own account, "will is manifested only where there is conscious, motivated or goal-oriented activity" (SV 200), an activity of a subject, a person, but an activity which takes place in time, and it is conceivable to have functions that would measure the level of consciousness, the level of motivation, and the level of activity. Even if these functions had some discontinuities of levels, they would be perfectly compatible with analytical processing. After all, Bugaev allowed for the possibility that "will manifests itself stronger when action is more energetic, livelier, more rational, more goal-oriented and freer, when is consciousness is wider and richer, the realm of self-consciousness is fuller and deeper, when motives of action are stronger, firmer and more general, i.e., when the personality of man opens up more strongly, more fully and more perfectly" (207) and it is possible to think about designing analytical functions measuring the level of energy, liveliness, etc. of actions.

Also, to tackle the problem of inverse function for functions which are not one-to-one, definitions of what could be termed enhanced inverses may be introduced, which would require the introduction of higher dimensions for ranges. For example, an enhanced inverse for $y = x^2$ would be the function $y \rightarrow (-\sqrt{y}, \sqrt{y}) \in R^2$, where R is the set of real numbers, i.e., a real number y would correspond to a unique ordered pair or real numbers, and thus, the square function that is of type $R \rightarrow R$, would have its enhance inverse of the type $R \rightarrow R^2$. For y

= sin(x), an enhanced inverse would be $y \rightarrow (\ldots, \arcsin(y)+2\pi, \arcsin(y)-\pi, \arcsin(y), \arcsin(y)+\pi, \arcsin(y)+2\pi, \ldots) \in R^Z$, where Z is the set of integers, and for $y = E(x)$, it would be $y \rightarrow [E(x), E(x)+1) \in \{[i, i+1): i \in Z\}$.

Bugaev stated that he did not reject transcendence, but in his discussions he wanted to remain in the realm of understanding accessible to all (SV 212). He even tried to downplay transcendence by saying that such transcendental problems as the problem of randomness of nature and the problem of necessity should have no influence on "the problems of the laws, morality, education, and society" (216). Such a philosophical simplification would be very difficult to defend. In fact, some hints of the transcendence may be detected in his statements. He did say that "if the world in its entirety can be viewed as a limitless individuality in which everything is orderly and rational, then man is the whole world in compressed and stereotypical edition." Science and life should "be able to read the meaning and significance of these two strange editions of the higher reason, learn from them to form themselves along with them in their actions. The essence of man and of the universe is hidden from us under impenetrable cover of profound secrecy" (215). Is the higher reason an allusion to God? Man and the universe are some editions of this high reason? Is it some circuitous way of following physico-theology by saying that God can be discovered and appreciated in His creation? Setting on equal footing the universe and a human being makes, in Bugaev's view, understandable "the wide evangelical statement, 'the Kingdom of God is in you' [Lk. 17:21]" (217). Although it is not easy to see why giving the same prominence to the world and to the human being in any way illuminates Christ's statement, Bugaev's arithmology was sometimes hailed as an avenue to a religious revival. For example, one author said that "when mechanics led us away from pursuing the Living God, then physiology of nature brings us back to Him. One branch of mathematics – calculus – killed faith, but other, higher branch – arithmology – leads to the reestablishment of faith worthy of the wise."[11]

In sum, mathematical arithmology in Bugaev's view encompassed the number theory and the analysis of discrete functions. His philosophical arithmology, on the other hand, was a worldview of the omnipresence of discontinuous phenomena. However, to scrutinize such phenomena, most of mathematics could be used including the classical calculus. Rather unaccounta-

[11] Mikhail O. Men'shikov, quoted in Алексеев, *op. cit.*, p. 27.

bly, Bugaev identified discontinuity with indeterminism as part of natural, social, and mental phenomena, and to that end it would also require to include the probability calculus and statistics. Thus, his mathematical arithmology would be only a minute part to tackle what he viewed as philosophical arithmology; the two arithmologies are thus widely misaligned.

Monadology

As mentioned, in Bugaev's view, discontinuity appears when there are self-standing individuals. There appears to be a conflict between continuity and individuality. He did say that when explaining events from the continuous point of view, laws of continuity are being determined and the individual is not taken into account. If it is, the explanation is from the discontinuous point of view.[12] The reasoning behind this statement may be that for a function $f(x)$, the individuality of an x is blurred with other individualities if x can be any value on the x-axis. However, if x can be only an integer, then its individuality clearly stands out against individualities of other x's. Be it as it may, Bugaev wanted to make the role of an individual being to philosophically stand out, unlike in the analytical worldview, which led him to building his version of monadology using Leibniz as the springboard.[13] He even retained the format of Leibniz' presentation: laconic numbered paragraphs not infrequently bordering on incomprehensibility.

"A monad is a living unit (единица), a living element. It is a self-standing and self-acting individual. It is *living* in the sense that it has a potential psychological content" (BM §§1-2), this content being observable by its outward manifestations and directly accessible only to the monad itself (§3). The active psychological content is "an immediate (intuitive) derivation, synthesis, sentence, conclusion, interpretation by the monad of inner factors of its being and of its

[12] Дополнительный протокол прений по поводу реферата Н. В. Бугаева "Основныя начала эволюционной монадологии", в заседании 7-го ноября 1892 г., *Вопросы философии и психологии* 4 (1893), p. 108.

[13] Bugaev expressed his high admiration for Leibniz. He devoted one article to his memory stating that "in Leibniz, a genius of a philosopher complements the genius of a mathematician"; he was "a great mathematician and a philosopher," Н. В. Бугаев, Общия основания исчисления Eφ(x) с одним независимым переменным, [part] IV, *Математический сборник* 13 (1887), no. 2, pp, 227-228.

relations to other monads" (§4). As a unit, the monad is constant; it does not change in some respects when change occurs. It is whole, indivisible, one, unchangeable, and a principle equal to itself in all relations to other monads. Life is a change that has to follow a certain law. The goal is the basis of this change; thus, life is an orderly goal-directed change. Life stems from inner causes called motives, reasons, incentives, and goals (§5). And again, "the life of a monad is a sequence of causal and goal-oriented changes in its organization" (§6). "The connection between psychological content and inner relations and manifestations corresponding to it depend on the laws that are above the monad and above its full understanding" (§7). "These laws not always can be discursively expressed in terms of words or concepts. Sometimes they are not submitted to quantitative relations" (§9). "Monads are of the first, second, third, etc. order" (§13). "Monads of the second order can form a monad of the first order" (§14). An example: the first order: man, the second order: cell, the third order: particle, the fourth order: atom (§15). Nation would be of the first higher order or "the monad of the minus first order" (§16).

Leibniz' monads are also self-standing, self-acting, self-developing entities (substances) (LM 18). In the departure from Leibniz whose monads were all on the same level, Bugaev required different levels of monads. However, on each level a monad is a monad, whether it is a society or an atom. Also, a monad is a living entity (BM §1) where life is expressed in terms of a psychological or mental content. Stronger yet, "at the basis of the nature of a monad is an active sensation or the will and the primal form of its psychological life is the impulse to existence and to the good [to be reached] through the active, self-executed and free development" (§152). Although in some metaphorical sense society can be viewed in terms of mental life, how can this be applied on lower-level monads, (confusingly, the lower a level, the higher the level number is)? Does a cell have any form of mental life? and an atom? and a sub-atomic particle?

There is an infinity of orders of monads (BM §17). However, does this infinity extend in both directions? From the society level monad we can go to, say, the planet level monad and then galactic level and then the entire universe. So, it does not appear that the hierarchy extends infinitely in the direction of super-monads (numbered with lower and lower negative integers).

In order to be able to go upward in the hierarchy of orders to lower and lower level of sub-monads, an infinite divisibility of matter must be assumed: atoms, subatomic particles, strings, etc. This assumption does not necessarily require that the makeup of matter is continuous. Infinite divisibility is possible in the discontinuous context. Consider an infinite process of bisecting an interval [*a*, *b*] which includes only rational numbers. The set is dense, but not continuous, so it is possible to view matter to have a similar structure, but that would only be an assumption, hardly testable.

The orders in the hierarchy of monads are determined by the composition of monads; however, this composition can be of two kinds: if a monad and its constituents do not significantly differ qualitatively and quantitatively, then the relation is of type: compound monad and component monads, like between a family and its members; if there is a significant qualitative and quantitative difference between a monad and its constitutive monads, then the relation is of the type: monad and sub-monads, as in the case of man and cells of the human body (BM §34). Compound monads are called complexes (§60) and there are complexes of different levels (§78) (complexes would correspond to Leibniz' aggregates of monads).[14] Also, there is a central or leading monad in each complex, so there is a micro-hierarchy inside each compound monad,[15] in which he followed Leibniz and his idea of a dominant monad (LM §70, *Principles of nature and grace, based on reason* §4).

Family considered as constituted of family members is a compound monad; on the other hand, society considered as constituted of society members is a monad composed of sub-monads. Would there be a monad of city dwellers, or street dwellers? It is possible that the street dwellers make a complex composed of people, city dwellers make a complex composed of street dweller complexes, and society would be a complex composed of city dweller complexes. However, if some of the levels of the hierarchy of complexes are skipped, we may get a relation of a different type, like between society and a person: a monad and a sub-monad. How many such levels need to be skipped for this to be possible? Can that even be determined? Consider a finer-grained hierarchy: street dwellers, suburb dwellers, city dwellers, county dwellers, state dwellers, etc. Moreover, if a person is a monad and a cell is a sub-monad, what about a bacterium? Is it

[14] Leibniz made this claim repeatedly, e.g., in a 1657 letter to Arnaud, in his *Philosophical essays*, Indianapolis: Hackett 1989, p. 85.

[15] It was suggested that Bugaev's distinction made between monads and complexes was supposed to make a distinction between the vertical order by the hierarchy of monads and horizontal order of compound monads, Шапошников, *op. cit.*, p. 81.

a monad with its single cell as a sub-monad? Is it a very uncomplicated complex? Is there any difference at all?[16]

It appears that durability can be introduced as a distinguishing feature between a monad and a complex: compound monads fall apart and constitute new complexes (BM §81); however, monads do not disappear, and complexes do not disappear either (§84). Bugaev's own example belies the validity of this distinction: humans do fall apart, at least their bodies, and so do cells of their bodies, and atoms do as well, some easily, and some must be forced to do it. Leibniz' required that monads, as simple substances, i.e., without parts, were imperishable (LM §3-4; *Theodicy* §396), well, except by the act of God (LM §6). Leibniz allowed for a complex organization of corporeal body – which was simply an infinity of incorporeal monads considered from the perceptual perspective – even to the extent that on each level of dividing matter, complex organization reappeared (LM §§67, 70; *Theodicy* §195), which, incidentally harks back to the everything in everything principle used by Anaxagoras. By requiring an infinite number of levels in the hierarchy of monads, Bugaev must assume the same: each monad consists of infinite layers of sub-monads. For admissibility of his view he must also allow monads to be permanently decomposable into their constitutive sub-monads: a person into cells, a cell into atoms, and the like, and thus, in effect, he has to allow for mortality of monads. Leibniz' monads are imperishable, but there is only one level of monads that can be reconfigured in many different ways, in each configuration one monad being in control of other monads, although not directly, but through the mechanism of the preestablished harmony (*Theodicy* §291).

And yet, Bugaev stated that a compound monad can fall apart, but the monad itself and the component monads do not disappear (BM §91); monads preserve their potential content which becomes active in favorable circumstances (§92); the compound monad continues its existence in the constitutive monads (§93), particularly in its central monad (§83). How does family, a compound monad, exist after the family breaks apart? Is the memory of this family in the minds of its members tantamount to the preservation of the existence of this family? And what about the case of memories that change and fade away over time? How does liver, a compound monad composed of cells, exist after it falls apart? Does its memory live on in the cells? Can these cells maintain their in-

[16] Cf. Timothy Langen, Nikolai Vasilievich Bugaev: a background, *Russian History* 38 (2011), p. 181.

tegrity after the liver ceases to exist as one functioning organ? It is not any different in the monad and sub-monad relation: when a person dies can the person-monad live on? We may assume that for this case Bugaev wanted to preserve the immortality of the soul in spite of the fact that somehow one monad would embrace a mortal body and an immortal soul. But the cell? An atom? How exactly do they continue to exist after they cease to exist as integral wholes?

Confusingly, however, Bugaev stated that a few simple monads can form a compound monad (BM §29). "Death is one of the transformational processes of the monad and it has only a relative meaning" (§166). "Simple monads never are born and never die" (§167). However, death is a process of transformation and it has only a relative meaning (§165) – by which we probably should understand that death is not annihilation – so, monads preserve their potential existence in new monadological forms (§168). What is a simple monad? Apparently the monad *tout court*, and thus Bugaev probably meant to say that monads (or simple monads) can form complexes as opposed to complexes forming complexes. Therefore, complexes die; monads that are not complexed don't by preserving their existence, if only potentially, in new monadological forms. What are these forms? Conceivable monadological forms would be monads, compound or simple. What would be a new form for an existing unbreakable monad? Bugaev required that "individuality and immortality of the monad are always preserved" (§118). Is this individuality preserved when the monadological form of the monad changes? Besides, never say never, and Bugaev's "never" should be qualified. He allowed monads to fall apart, after all; however, "a monad can disintegrate into sub-monads only in the cases of special permission and plan sometimes going beyond conditions accessible and understandable to us" (§35), conditions being "the laws to which the monad is submitted" (§8). Disintegration of cells into atoms is hardly an extraordinary event and hardly taking place according to the laws which the human mind cannot comprehend. Could Bugaev have meant here the possibility of annihilation of the soul?

Monads develop themselves through their own effort and through their drive to unification – creating societies of monads (BM §120) – they develop habits (§§121, 128, 135), good habits, that is, and the most widespread and simplest of them become physical laws of nature (§123). Would that mean that the law of gravitation emerges at a certain level of the development of monads and establishes itself through their effort but that there was none on earlier stages of the history of the world? Apparently (cf. §125). Would that mean that a dropped heavy object on earlier stages of the history of nature could just as well fall down, move sideways or even float?

The basis of the life and activity of a monad is perfecting itself and other monads (BM §100) to reach the level when its component monads "express the best way the idea of the whole" (§102). When people share the same views, as are held in the society, then social cohesion is improved, which may be considered a more perfect level for the development of the society and thus this may be considered an expression of the idea of the whole. What if the ruling views of the society are nefarious? Would a popular consent with these views be considered a sign of the perfection of society? Also, how are atoms supposed to share and then express the idea of the whole cell? Would the mental life of an atom be sufficient for such an expression to be possible?

"The world is the collection of a huge number of simple and compound monads of various orders" (BM §146). "The life of the world consists in constant process of forming and reforming compound monads under the influence of the drive of simple monads and compound monads to the mutual perfecting with the help of ethical laws of rise and growth" (§147; cf. §§51-52) in order to increase the mental content of the monads to the level of the mental content of the entire world and to make the entire world a monad (§148). The world, thus, consists of hierarchically ordered monads and complexes, but somehow the world itself is not a monad even though it does have some form of mental life. Does the world have to be made a simple monad? However, they are never born, as already stated (§167). Also, including complexes, the world can at best be a complex. Would being a compound monad be good enough for the world? Yet compound monads are transitory. Apparently, the world would also be transitory. Maybe because of that it was possible for Bugaev to say that the ultimate goal of the activity of a monad is to remove the difference between the monad and the world as the collection of all monads, to reach the infinite perfection and stand over the world (§104), and again, a distant goal of a monad is an effort to stand outside the world or over the world by making itself beforehand the world or through the world (§111). The monad – be it a person or an atom – can conceivably elevate itself above the level of the world, thereby having its perfection succeed the perfection of the world. However, what would it do to the hierarchy of monads? Perfection of a monad is better than the perfection of the world; wouldn't the position of this monad be improved in the hierarchy of monads?

Leibniz' monads were without windows (LM §7); they were worlds to themselves. All communication between monads, even with their own bodies (§78), was the result of the pre-established harmony. If they influence one another, it is done indirectly through the mediation of God (§51). Bugaev's monads do cooperate (BM §26, 28, 39) through solidarity (§113) which is love (§114); in fact, the monad cannot change its mental content by itself outside a relation with order monad(s) (§68); they can improve themselves only by establishing a relationship with other monads (§70). Monads do evolve and for this reason Bugaev called his theory the evolutionary monadology. And so, only by their activity can monads improve their mental content (§53) which is done by teaching and learning (§55); however, a compound monad is a condition of progress of the monads that constitute it (§59). And yet, Bugaev in the introduction to his monadology article said that his theory bears some similarity to "the monadology of the pre-established harmony." However, pre-established harmony requires a being who pre-establishes this harmony and the presence of God in Leibniz is very strong; in Bugaev, on the other hand, it is barely discernible. He assured his readers that his monadology is based on science and "goes hand in hand with the tasks of ethics, sociology, and with all most profound teachings about the Unconditional" (§178). Also, "the real essence and provenance of the monad is explained not by a philosophical system, but by profound teaching about the Unconditional" (§157). Not even a hint about this profound teaching. A resort is made, if only fleetingly, to the Unconditional.

The evolution of the world is progressive, it improves the world, it aims at the oneness of the world. The goal-orientation is part of the makeup of each monad. How is it, that all monads have the same goal? Did they all somehow on their own – humans, cells, atoms, etc. – figure out first that the best course of action is unity instead of, say partial unity – tribes, nations, but also, separate molecules and organisms?[17] Or was particular goal-orientation stamped already on their nature – stamped by the Unconditional? On the other hand, assuming that the worldwide unity will be created, what will be the position of the Unconditional? A separate monad? Since Bugaev claimed that his system united pantheism and individualism (§119), would the Unconditional be blended with

[17] Cf. Л[ев] М. Лопатин, Философское мировоззрение Н.В. Бугаева, *Вопросы философии и психологии* 15 (1904), no. 72 (II), p. 183.

the world having observed and, possibly, providentially assisted the world in its unification process?

Nekrasov tried to fill the gap by saying that the evolution of the world process is directed by "the Unconditional Being, i.e., God"[18] and that the world was created by "the Unconditional Being (the triune God)."[19] God determined the worldwide ethics or truth which becomes an element of "the pre-established *world* ideal-real harmony."[20] Nekrasov's book-long article is full of religious references and he apparently tried to reconcile Bugaev's monadology with Christianity when speaking about the triune God. That would be quite a difficult task. Bugaev's monads are not born nor do they die. Would they be co-eternal with God, which would be something akin to the world of Aristotle? Christians believe that Christ is born of God and yet eternal. Could Bugaev venture to say that it would be something similar with all monads? Since improvement in Bugaev's world depends solely on the activity of the monad, what would be the role of salvific work of Christ in this framework? What about some elements of the Orthodox faith. For example, Leibniz struggled to reconcile the doctrine of transubstantiation with his monadology.[21] Could it be reconciled with Bugaev's version?

The unity of the world as the end of progress resembles the goal spelled out by Leibniz, which for him was the unity of spirits in the city of God, the most perfect order that can be, a moral order within the natural order (LM §85-86), the wonders and greatness of which were revealed to humans by Christ (*Discourse on metaphysics* §37). It also resembles the goal of the cosmic development discussed by Vladimir Solovyov.[22] First, Solovyov spoke about eternal and immutable atoms that are immaterial and they are elementary forces of reality.[23] These forces are conscious, whereby they are living entities or monads (52).

[18] Некрасов, *op. cit.*, pp. 104, 107.

[19] Некрасов, *op. cit.*, pp. 105, 159-160.

[20] Некрасов, *op. cit.*, p. 106.

[21] Leibniz, A 1712 letter to Father des Bosses, in *Philosophical essays*, p. 198.

[22] Bugaev knew Solovyov well. In 1874 he voted for Solovyov to be included in the institute of philosophy in Moscow С[ергей] С. Демидов, Н. В. Бугаев и возникновение Московской школы теории функций действительного переменного, *Историко-математические исследования* 29 (1985), p. 117; he also "recognized talent in the philosopher Solovyov," Андрей Белый, *На рубеже двух столетий*, Москва: Художественная Литература 1989, p. 369. It is possible, that Bugaev was introduced to Leibniz in 1874 through the first book of Solovyov, *The crisis of Western philosophy*, Hudson: Lindisfarne Press 1996 [1874], pp. 25-28.

[23] Vladimir Solovyov, *Lectures on divine humanity*, Hudson: Lindisfarne Press 1995 [1877-1881], pp. 48-49.

Each atom has an immutable quality, an idea that determined it actions (50-51). He also spoke about a hierarchy of organisms of entities (ideas), one organism forming organisms from a lower level and each organism possessing a specific idea, although the hierarchy did not appear to extend infinitely. The highest organism possesses the absolute love (53). Before creation, all entities were in God, who is the all-one. Creation is allowing these entities to live autonomously (128-130). The all-unity was maintained by the world-soul (131), but the world-soul separated itself from God, whereby the natural world emerged (133-134). The disorder of this natural world manifests itself as physical matter (124); that is, matter is really an illusory aspect of reality. Restoration of order takes place through the gradual restoration of unity (139) in which the Incarnation of Christ plays the critical role (157-158, 161).

Bugaev wanted to reconcile in his monadology the spirit and matter (BM §119) considering "matter and spirit to be consequences derived from two forms of the relation of one monad to another" (§132): disregarding the monad's inner life leads to its material view, concentration on its inner life leads to its spiritual view, to spiritualization of matter (§133) which makes his monadology ontologically a form of monism and epistemologically a form of dualism.[24] Monism was also on Leibniz' mind by making monads immaterial and their bodies to be composed of immaterial monads; thus, physical matter is but a way of perceiving the immaterial.[25]

Bugaev proposed his monadology as a philosophical way to combat the predominance of analytical worldview in science and in philosophy. As he phrased it, "the idea of continuity of natural phenomena started to penetrate to biology, psychology, and sociology. The teachings of Lamarck and Darwin are nothing else than an attempt to apply in biology to continuous change of phenomena the views which rule in geometry, mechanics, and physics" (MiN 706). It is rather ironic that by naming his monadology evolutionary, he almost established a dependence between his philosophy and Darwin.[26] In fact, his

[24] As reported by his son, his monadology was characterized by its "dialectical reality," Белый, *op. cit.*, p. 172.

[25] "Matter isn't composed of constitutive unities, but results from them, since matter, that is, extended mass is only a phenomenon grounded in things, like a rainbow or a parhelion, and all reality belongs only to unities," Leibniz, A 1704 letter to Volder, in his *Philosophical essays*, p. 179; "it may be said that there is nothing in things but simple substances, and in them, perception and appetite. Moreover, matter and motion are not so much substances or things as they are the phenomena of perceivers," p. 181.

[26] This evolutionism was supposed to be different than the evolutionism of Spencer, Белый,

monadology is perfectly compatible with Darwinism and, more generally, with analytical thinking which models natural and social phenomena with continuous functions. Monads and their complexes are, to be sure individual entities, but they develop in time, even if in this development jumps may occur from one level of progress to another instead of continuous transitions. Darwinism could accept occasional jumps in the development of species without relinquishing the analytical mode of thinking – consider the punctuated equilibrium view incorporated in Darwinism. Therefore, Bugaev did not offer anything significantly new in respect to establishing the arithmological paradigm; he just proposed a version of monadology somewhat different from Leibniz' and Solovyov's, which rather well fits the analytical worldview.

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op. cit., p. 172; supposedly because of possible discontinuities between different levels of development since the process of development itself is continuous (§42).

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