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A DECISION RULE FOR UNCERTAIN MULTI-CRITERIA PURE DECISION MAKING AND INDEPENDENT CRITERIA[1]

Summary

The paper is concerned with multi-criteria decision-making under uncertainty with scenario planning. This topic has been explored by many researchers since almost all real-world decision problems contain multiple conflicting criteria and a deterministic evaluation of criteria is often impossible. We propose a procedure for uncertain multi-objective optimization which can be applied when seeking a pure strategy. A pure strategy, as opposed to a mixed strategy, allows the decision-maker to select and perform only one accessible alternative. The new approach takes into account the decision-maker's preference structure (importance of particular goals) and nature (pessimistic, moderate or optimistic attitude towards a given problem). It is designed for one-shot decisions made under uncertainty with unknown probabilities (frequencies), see decision-making under complete uncertainty or decision-making under strategic uncertainty. The novel approach can be used in the case of totally independent payoff matrices for particular targets.

Keywords: uncertainty, multi-criteria decision-making, pure strategies, one-shot decisions, independent criteria, two-stage models.

JEL: C44, D81, L21.

1. Introduction

Multiple criteria decision-making with uncertain attribute (criterion) evaluations has been theoretically and practically investigated by many researchers since usually real decision problems contain numerous conflicting criteria and a deterministic evaluation of criteria is often impossible. [Durbach and Stewart 2012] prepared an impressive review of possible models, methods and tools supporting uncertain multi-criteria decision-making (e.g. models with explicit risk measures, models with scenarios, models with fuzzy numbers, models using probabilities or probability-like quantities) [Gaspars-Wieloch, 2015c]. In this paper we propose a method designed for multi-criteria decision- making with scenario planning and one-shot decision problems (we assume that

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after the realization of a selected alternative the decision-maker may change his/her preferences – thus, multi-shot decisions are not considered). We analyze the case of games against nature (not games with other players). We assume that criteria payoff matrices are independent and that, within each criterion, payoffs connected with a given decision constitute sequences of outcomes (not sets of outcomes). We will refer to a multi-criteria two-stage model. The target of the new approach is to select an optimal pure strategy, which means that only one alternative is selected and performed. The procedure takes into consideration decision-makers' objective preferences and their attitude towards the risk connected with a given problem. This attitude is measured by the coefficient of optimism on the basis of which a set of events having the biggest subjective chance of occurrence (separately for each payoff matrix) is suggested. Such an approach allows us to reduce the quantity of data considered in the final decision selection.

The paper is organized as follows. Section 2 deals with the main features of MDMU (multi-criteria decision-making under uncertainty), scenario planning, 1-stage and 2-stage models. Section 3 presents a procedure that can be used as a tool in multi-criteria optimization under uncertainty for pure strategies and independent payoff matrices. Section 4 provides a case study on the basis of the bi-criteria single-period newsvendor problem. Conclusions are gathered in the last Section. The paper is a continuation of several articles, where uncertain one-criterion procedures [Gaspars-Wieloch, 2007; 2014a; 2014c; 2015b; 2016b] and multi-criteria decision rules [Gaspars-Wieloch, 2014d; 2015c; 2015d; 2017] are investigated.

2. How to combine criteria with scenarios in uncertain multi-criteria decision making when criteria are independent?

The necessity to solve decision problems with uncertain parameters led to the development of many diverse theories [Zio, Pedroni, 2013], e.g. probability theory [Kolmogorov, 1933], possibility theory [Zadeh, 1978; Dubois, Prade, 2001], imprecise (interval) probability [Walley, 1991], uncertainty theory [Liu, 2007; 2009], credibility theory, evidence theory [Shafer, 1976; Sentz, Ferson, 2002]. Nevertheless, it is worth emphasizing that there is no unanimity in defining the notion of uncertainty and that there are many types of uncertainty. The three next paragraphs are prepared on the basis of [Gaspars-Wieloch, 2016a; 2017].

According to the theory of decision the decision-maker (DM) may choose the appropriate alternative (decision, strategy, variant) under certainty (DMC – parameters are deterministic), under risk (DMR – possible scenarios and their likelihood are known), with partial information (DMPI – possible states of nature are known, but their probability is not completely known), under complete uncertainty (DMCU – scenarios are known, but not the probability of their occurrence) or under total ignorance (DMTI – the DM is not able to define possible events). Note that DMCU may also occur when the DM does not want to make use of the estimated probability distribution [Trzaskalik, 2008]. Comments concerning particular decision-making circumstances can be found for instance in [Guo, 2011; Kaplan, Barish, 1967; Knight, 1921; Perez et al., 2015; Render et al., 2006;

Sikora, 2008; Waters, 2011; Weber, 1987]. Uncertainty and risk were formally integrated in economic theory by [von Neuman and Morgenstern 1944].

Supporters of the theory of economics state that uncertainty involves all situations with non-deterministic parameters (known, unknown or incompletely known probability distribution, lack of information about possible scenarios), while risk is related to the possibility that some bad (or other than predicted) circumstances will happen [Dominiak, 2009; Dubois, Prade, 2012; Fishburn, 1984; Gaspars-Wieloch, 2016a; Guney, Newell, 2015; Ogryczak, Sliwinski, 2009; Waters, 2011]. Note that in some cases the exact probability computation may be difficult due to: 1) the existence of diverse definitions of probability [Carnap, 1950; De Finetti, 1975; Frechet, 1938; Hau et al., 2009; Knight, 1921; Kolmogorov, 1933; Piegat, 2010; Popper, 1959; Ramsey, 1931; Van Lambalgen, 1996; Von Mises, 1949; Von Mises, 1957], 2) the lack of historical data (for totally new decisions and events) [Gaspars-Wieloch, 2016a; Guo, 2011; Guo, Ma, 2014], 3) the lack of sufficient knowledge about particular states or 4) the fact that the set of possible scenarios forecasted by experts in the scenario planning stage does not satisfy probability axioms (the sum of state probabilities should be equal to 1, the whole sample space must be precisely defined), see [Kolmogorov, 1933]. Caplan [2001] states that sometimes people may be even unable to declare subjective probabilities, but they implicitly set the likelihood in acting. Nevertheless, within the theory of economics, even if the probability is not known, some probability-like quantities can often be estimated and applied. Hence, uncertainty, in many cases, can be measured and quantified somehow [Piasecki, 2016].

Besides two aforementioned approaches, it is worth mentioning the Austrian Economic School which treats uncertainty as do decision theorists, i.e. as a situation where the likelihood is not known. Additionally, it is assumed that the mathematical probability of the occurrence of a given scenario is not known as probabilities only concern repetitive events, meanwhile in the majority of real problems the DM deals with non-repetitive events. According to [Von Mises, 1949], the theory of probability can never lead to a definite statement concerning a single event (the probability of a single event cannot be presented numerically). Uncertainty is not caused by the randomness of events (as held by mainstream economists) but is due to numerous factors, of which only some are known in the decision-making process [Gaspars-Wieloch, 2017].

Note that scientists distinguish two main types of uncertainty: the epistemic (reducible) uncertainty – due to the lack of knowledge (it can be reduced or eliminated after collecting information), and the aleatory (aleatoric, random) uncertainty – due to the inherent variability in a physical phenomenon (it cannot be reduced even after conducting n experiments) [Stirtling, 2003; Zio, Pedroni, 2013].

In this paper we consider both epistemic and aleatory uncertainty, which leads us to a conclusion that the likelihood of the occurrence of particular states of nature cannot be estimated in an accurate way (the aleatory uncertainty is not reducible). Additionally, in connection with the fact that the contribution concerns only one-shot decisions, we refer in a sense to the Austrian approach where the probability understood as frequency cannot be computed for a single event. The theory of economics is also partially applied in this research since unknown probabilities are replaced with some secondary probability-like quantities.

It is worth adding that, as a matter of fact, scientists declare different opinions concerning the role of probability in scenario planning (SP). Some of them state that the likelihood should not be applied to SP [Michnik, 2013]. Others are convinced that there are many advantages of using probabilities in SP [Millett, 2009].

The notion "uncertainty" has been just briefly discussed. Now, let us analyze possible ways of inserting uncertainty in multi-criteria decision-making. Many classical and extended decision rules designed for pure or mixed MDMU (multi-criteria decision-making under uncertainty) have been already developed, e.g. [Aghdaie et al., 2013; Ben Amor et al., 2007; Dominiak, 2006; 2009; Durbach, 2014; Eiselt, Marianov, 2014; Gaspars-Wieloch, 2014d; 2015c; 2015d; 2017; Ginevičius, Zubrecovas, 2009; Goodwin, Wright, 2001; Hopfe et al., 2013; Janjic et al., 2013; Korhonen, 2001; Lee, 2012; Liu et al., 2011; Lo, Michnik, 2010; Michnik, 2013; Mikhaidov, Tsvetinov, 2004; Montibeller et al., 2006; Ram et al., 2010; Ramik et al., 2008; Ravindran, 2008; Silva, 2016; Stewart, 2005; Suo et al., 2012; Troutt, Pettypool, 1989; Tsaur et al., 2002; Urli, Nadeau, 2004; Wojewnik, Szapiro, 2010; Xu, 2000; Yu, 2002]. Some of them can be applied when the DM intends to perform the selected alternative only once (one-shot decisions). Others are recommended for people considering multiple realizations of the chosen variant (multi-shot decisions). In the aforementioned contributions diverse tools connected with uncertainty are used, e.g. probabilities, belief functions, fuzzy membership functions. However [Durbarch, Stewart, 2012] underline that uncertainties become increasingly so complex that the elicitation of such measures becomes operationally difficult for DMs to comprehend and virtually impossible to validate. Therefore, in their opinion, it is useful to construct scenarios which describe possible ways in which the future might unfold and to combine MDMU with SP (scenario planning). And then, the result of the choice made under uncertainty with scenario planning depends on two factors: which decision will be chosen and which state of nature will occur [Gaspars-Wieloch, 2015c].

According to [Durbarch, Stewart, 2012; Michnik, 2013] MDMU+SP models can be divided into two classes (the description below has been prepared on the basis of [Gaspars-Wieloch, 2014d; 2015c; 2015d; 2017]). The first one (A) includes 2-stage models in which evaluations of particular alternatives are estimated in respect of scenarios and criteria in two separate stages. Class A contains two subclasses: A-CS and A-SC. Subclass A-CS denotes the set of approaches considering decisions separately in each scenario and setting a $n \times m$ table (n – number of decisions, m – number of scenarios) giving the aggregated (over attributes/criteria) performance of alternative D_j under scenario S_i. These evaluations are then aggregated over scenarios. In subclass A-SC the order of aggregation is reversed – performances are generated across scenarios and measures are then calculated over criteria. The second class (B) consists of one-stage procedures considering all combinations of scenarios and attributes (scenario-criterion pairs) as distinct meta-criteria and using a chosen multiple criteria approach for the transformed meta-matrix. There is currently no consensus on the best way to solve uncertain multi-goal problems [Durbarch and Stewart 2012]. Let us emphasize that subclass A-CS may be only applied to dependent payoff matrices. Hence, the number of events ought to be the same for each criterion considered in the decision problem and evaluation a_{ij}^k can only be connected with evaluations a_{ij}^1, …, a_{ij}^{k-1}, a_{ij}^{k+1},…, a_{ij}^{p-1} and a_{ij}^p (those values describe the performance of each criterion by decision D_j provided that scenario S_i happens) where p is the number of criteria. On the other hand, subclass A-SC should merely be used for independent payoff matrices, which means that this time there is no relationship between criteria. The performance of particular targets can be analyzed totally separately since the number of states of nature can be different for each goal (m_1, m_2, …, m_k, …, m_p). In the second case, evaluation a_{ij}^k might be connected with any evaluation a_{ij}^1 ($i = 1, …, m_1$), any evaluation a_{ij}^2 ($i = 1, …, m_2$), … and any evaluation a_{ij}^p ($i = 1, …, m_p$). Those values describe the performance of each criterion by decision D_j assuming that any scenario occurs for criteria C_1, …, C_{k-1}, C_{k+1},…, C_p . One-stage models (i.e. class B) are also designed for independent payoff matrices.

Subclass A-SC from two-stage models, combined simultaneously with uncertain multi-criteria pure decision-making, games against nature, probability-like quantities, scenario forecasting stage and independent criteria, hasn't so far been analyzed in the literature. Nevertheless, we would like to investigate this topic, since it gives us the possibility to elaborate a procedure focusing on events having the biggest subjective chance of occurrence. Note that the set of aforementioned scenarios will not be directly predicted by the DM – it will be indirectly generated on the basis of his/her coefficient of optimism.

3. Procedure for MDMU+SP, pure strategies and independent criteria

The discrete version (i.e. the set of alternatives is explicitly defined and discrete) of MDMU+SP with independent payoff matrices consists of n decisions (D_1, …, D_j, …, D_n), each evaluated on p criteria (C_1, …, C_k, …, C_p) and m_k mutually exclusive scenarios (S^k_1, …, S^k_i, …, S^k_{mk}) where $k = 1, …, p$. The problem can be presented by means of p payoff matrices (one for each criterion) and $n\times(m_1+…+m_k+…+ m_p)$ evaluations. Each payoff matrix contains $n\times m_k$ evaluations, say a_{ij}^k, which denotes the performance of criterion C_k resulting from the choice of decision D_j and the occurrence of scenario S^k_i. We assume that the distribution of payoffs related to a given decision is discrete.

Instead of using probabilities, we will apply here the coefficients of optimism (β) and pessimism (α). They allow us to take into account the DM's nature (attitude towards a given problem) and to generate some secondary probability-like quantities. These parameters belong to interval [0,1] and satisfy the condition $\alpha+\beta=1$, where α (β) tends to 0 (1) for extreme optimists (risk-prone behavior) and is close to 1 (0) for radical pessimists (risk-averse behavior). The coefficients of pessimism and optimism have been already used in decision rules suggested for instance by [Gaspars-Wieloch, 2014b; 2015a; Hurwicz, 1952; Perez et al., 2015].

The procedure presented in this Section considers DM's preferences (criteria weights) and suppositions (optimism/pessimism level, indirect choice of the scenarios with the biggest subjective chance of occurrence).

The suggested method (based on subclass A-SC) consists of the following steps:
1) Given a set of potential decisions and payoff matrices for each criterion, define an appropriate value of parameter $\beta \in [0,1]$ according to your level of optimism, and choose weights w^k for each attribute (k=1,…,p). The weights ought to describe the importance of each target and should satisfy Equation (1):

$$\sum_{k=1}^{p} w^k = 1 \tag{1}$$

2) If necessary (i.e. when criteria are presented in different scales or units), normalize the evaluations (use Equation (2) for maximized criteria and Equation (3) for minimized criteria) separately within each payoff matrix:

$$a(n)_{ij}^k = \frac{a_{ij}^k - \min\limits_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \{a_{ij}^k\}}{\max\limits_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \{a_{ij}^k\} - \min\limits_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \{a_{ij}^k\}} \qquad i = 1,\dots,m_k;\ j = 1,\dots,n;\ k = 1,\dots,p \tag{2}$$

$$a(n)_{ij}^k = \frac{\max\limits_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \{a_{ij}^k\} - a_{ij}^k}{\max\limits_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \{a_{ij}^k\} - \min\limits_{\substack{i=1,\dots,m\\ j=1,\dots,n}} \{a_{ij}^k\}} \qquad i = 1,\dots,m_k;\ j = 1,\dots,n;\ k = 1,\dots,p \tag{3}$$

where $a(n)^k_{ij}$ denotes the normalized evaluation of a^k_{ij}. Expressions max$\{a^k_{ij}\}$ and min$\{a^k_{ij}\}$ signify the highest and the lowest evaluation of criterion C_k in payoff matrix related to this attribute.
3) Compute the sum of cumulative relative profits ($p(r)^k_i$) for each scenario (separately for each criterion).

$$p(r)_i^k = \sum_{j=1}^{n} p(r)_{ij}^k \qquad i = 1,\dots,m_k;\ k = 1,\dots,p \tag{4}$$

$$p(r)_{ij}^k = m_k \cdot a(n)_{ij}^k - \sum_{i=1}^{m_k} a(n)_{ij}^k \qquad i = 1,\dots,m_k;\ j = 1,\dots,n;\ k = 1,\dots,p \tag{5}$$

This step allows identifying scenarios that are relatively better than others. In the contribution, the dominance is measured by means of the distance between particular evaluations related to a given scenario and all corresponding values connected consecutive decisions. The method presented above is just a suggestion. The status of each event may be estimated in a different way.
4) Choose the set of scenarios with the biggest chance of occurrence for each criterion separately:

a) If there exists at least one scenario for which the sum of relative profits is exactly equal to the value given by Equation (6), the set of events with the biggest chance of occurrence (SS^{k*}) contains all such states of nature (the number of scenarios belonging to that set is denoted by m_{k^*}). Go to step 5a.

$$p(r)_{\beta}^{k} = \beta \cdot \left(p(r)_{\max}^{k} - p(r)_{\min}^{k}\right) + p(r)_{\min}^{k} \qquad k = 1,\ldots,p \tag{6}$$

$$p(r)_{\max}^{k} = \max_{i}\left\{p(r)_{i}^{k}\right\} \qquad k = 1,\ldots,p \tag{7}$$

$$p(r)_{\min}^{k} = \min_{i}\left\{p(r)_{i}^{k}\right\} \qquad k = 1,\ldots,p \tag{8}$$

b) If there is no scenario fulfilling Equation (6), find all events $S^{k,\beta}_{min}$ satisfying condition (9) – the set containing such states of nature is denoted by $SS^{k*}_{\min}$ and the cardinality of that set is defined by $m^{\min}_{k*}$, and all events $S^{k,\beta}_{\max}$ satisfying condition (10) – the set containing such scenarios is denoted by $SS^{k*}_{\max}$ and its cardinality is defined by $m^{\max}_{k*}$. All these states of nature constitute the set of scenarios with the biggest chance of occurrence (i.e. SS^{k*}): $SS^{k*}_{\min} \cup SS^{k*}_{\max} = SS^{k*}$. Go to step 5b.

$$S_{\min}^{k,\beta} = \arg \min_{S_i^k \mid p(r)_i^k < p(r)_\beta^k} \left(p(r)_{\beta}^{k} - p(r)_{i}^{k}\right) \qquad k = 1,\ldots p \tag{9}$$

$$S_{\max}^{k,\beta} = \arg \min_{S_i^k \mid p(r)_i^k > p(r)_\beta^k} \left(p(r)_{i}^{k} - p(r)_{\beta}^{k}\right) \qquad k = 1,\ldots p \tag{10}$$

Hence, we see that the choice of scenarios with the biggest change of occurrence strictly depends on the level of β. We assume that the higher the value of β, the better scenarios (i.e. with higher sums of cumulative relative profits) should be treated as events reflecting the DM's nature. Step 4b involves situations where there is no event exactly fitting the level of the coefficient of optimism (according to Equation (6)). In such circumstances, one can for instance find scenarios for which the sum of cumulative relative profits is very close to the value indicated by Equation (6).

5) a) Calculate average values for each decision according to Equation (11), separately for each criterion.

$$v_{j}^{k} = \frac{1}{m_{k*}} \cdot \sum_{S_i^k \in SS^{k*}} a(n)_{ij}^{k} \qquad j = 1,\ldots,n; k = 1,\ldots,p \tag{11}$$

b) Calculate weighted values for each decision according to Equation (12), separately for each criterion.

$$v_{j}^{k} = \frac{1}{m_{k*}^{\max}} \cdot \sum_{S_i^k \in SS_{\max}^{k*}} a(n)_{ij}^{k} \cdot \frac{p(r)_{\beta}^{k} - p(r)_{\min}^{k*}}{p(r)_{\max}^{k*} - p(r)_{\min}^{k*}} + \frac{1}{m_{k*}^{\min}} \cdot \sum_{S_i^k \in SS_{\min}^{k*}} a(n)_{ij}^{k} \cdot \frac{p(r)_{\max}^{k*} - p(r)_{\beta}^{k}}{p(r)_{\max}^{k*} - p(r)_{\min}^{k*}}$$

$$j = 1,\ldots,n; k = 1,\ldots,p \tag{12}$$

In this step all evaluations connected with a given alternative and related to set SS^{k*} are aggregated in order to generate only one index for each decision within each criterion. In step 5a the weight of each value is the same since all of them come from scenarios with the same level of the sum of cumulative relative profits. On the other hand, in step 5b weights for particular values may be different as the differences between sums of cumulative relative profits and the theoretical one (defined by Equation (6)) may be diverse, depending on the scenario. The aggregation performed in step 5 completes the first stage of the 2-stage model.

6) Compute a weighted index for each decision across all criteria, using the SAW method (Simple Additive Weighting Method).

$$SAW_j = w^k \cdot v_j^k \qquad j = 1,\ldots,n \tag{13}$$

7) Choose the final pure strategy:

a) Find the alternative fulfilling Equation (14).

$$D_{j^*} = \arg\max_j \left(SAW_j\right) \tag{14}$$

b) If D_{j^*} satisfies condition (15) for the criterion with the highest weight w^k, D_{j^*} is the optimal pure strategy.

$$\forall_{t \in \{1,2,\ldots,z_k\}} \left(a(n)_{t,j^*} \geq wald_{best}^k\right) \qquad k \mid w^k = \max_k \{w^k\} \tag{15}$$

$$z_k = \lceil (1-\beta) \cdot m_k \rceil = \lceil \alpha \cdot m_k \rceil \qquad k \mid w^k = \max_k \{w^k\} \tag{16}$$

$$a(n)_{1,j^*} \geq \ldots \geq a(n)_{t,j^*} \geq \ldots \geq a_{z_k,j^*} \tag{17}$$

$$wald_j^k = \min_i \left\{a(n)_{ij}^k\right\} \qquad j = 1,\ldots,n; k \mid w^k = \max_k \{w^k\} \tag{18}$$

$$wald_{best}^k = \max_j \left\{wald_j^k\right\} \qquad k \mid w^k = \max_k \{w^k\} \tag{19}$$

where z_k is the minimal number of scenarios for which normalized outcomes should be at least equal to $wald^k_{best.}$ The last symbol denotes the highest value of Wald index for the criterion with the highest weight w^k.

c) If D_{j^*} does not fulfill condition (15), find $D_{j^{**}}$ satisfying Equation (20). Note that such a decision always exists.

$$\left(SAW_j \to \max\right) \wedge \left(\forall_{t \in \{1,2,\ldots,z_k\}} \left(a(n)_{t,j} \geq wald_{best}^k\right)\right) \qquad k \mid w^k = \max_k \{w^k\} \tag{20}$$

Step 7 also requires additional explanations. As a matter of fact, we could have stopped the algorithm after step 7a. Nevertheless, the choice of the final alternative merely on the basis of Equation (14) may be unfair especially for moderate and radical pessimists who feel more safely when the procedure provides necessary securities. For a radical pessimist, Equations (15)-(20) guarantee that even if the worst scenario connected with the selected decision and the most important criterion occurs, the outcome related to this goal will not be lower than the Wald index computed for the aforementioned target. Similar securities, but to a lesser extent, are also guaranteed for other types of DMs (moderate pessimists, moderate optimists etc.).

Let us name the procedure presented above β-MPDM/2, i.e. a method referring to parameter *β*, designed for multi-criteria pure decision-making and based on two-stage models. The choice of the set of events having the biggest subjective chance of occurrence (step 4) results from the fact that in the case of one-shot decisions only one scenario (within each criterion) will finally occur in the future. The use of the SAW method in step 6 is just a suggestion. Other multi-criteria approaches are also possible [Trzaskalik, 2014; Wachowicz, 2015].

4. Case study

The novel approach will be illustrated by means of an example concerning the bi-criteria single period newsvendor problem. The one-criterion problem is described, e.g. in [Gaspars-Wieloch, 2016a; Sikora, 2008]. The newsvendor problem (NP) has attracted a great deal of attention and played a central role at the conceptual foundations of stochastic inventory theory [Bieniek, 2016]. It was originally related to decision making under stochastic uncertainty where the demand is presented as a random variable with a known probability distribution. Nevertheless, NP has also been recently discussed in the context of decision making with partial information [Guo, 2011; Guo, Ma, 2014], where the DM is able to subjectively define possibility degrees and satisfaction levels (the probability distribution is not known completely). Additionally, according to [Besbes, Muharremoglu, 2013; Benzion et al., 2010; Gaspars-Wieloch, 2016a], the newsvendor theory should not assume that the DM faces a known distribution (known frequencies), since in real-life situations the demand distribution is not always known (e.g. for innovative products where there are no data available for forecasting the upcoming demand via statistical analysis).

Les us assume that the newsvendor intends to sell a totally new short-cycle product. He assumes that the quantity procured will be solely used to satisfy the demand during the current period. The demand for this product is not known in advance. He considers order (q) and demand (D) quantities between 1 and 5 boxes. Hence, there are 5 alternatives: A_1 (1 box), A_2 (2 boxes), A_3 (3 boxes), A_4 (4 boxes) and A_5 (5 boxes). The unit production/ purchase cost of 1 box (c_1) equals 5, the selling price (c_2) equals 10 and the discount price (price of leftover items) c_3=1, hence the unit profit from selling the product at price c_2: $b=c_2-c_1=5$ and the unit loss from selling it at price c_3: $s=c_1-c_3=4$. The newsvendor maximizes the total profit (e.g. in thousands of Euros) resulting from buying and selling the new product (the 1st criterion depends on the demand – five possible scenarios) and minimizes the cost of supply (also in thousands of Euros, the 2nd criterion depends on the supplying, storage, weather conditions – three possible scenarios). Note that the total profit does not include the cost of supply and that is equal to $b\times q$ (for $q\leq D$) or $b\times D-s\times(q-D)$ when $q>D$. Payoff matrices are given in Tables 1-2 (first value in each cell). The newsvendor intends to find an optimal pure strategy. Now, let us apply the procedure β-MPDM/2 for the aforementioned problem.

TABLE 1.

Payoff matrix and normalized values (1st criterion)

Crit. 1	$A_1 = 1$	$A_2 = 2$	$A_3 = 3$	$A_4 = 4$	$A_5 = 5$
$S^1{}_1 = 1$	5/0.44	1/0.33	-3/0.22	-7/0.11	-11/0.00
$S^1{}_2 = 2$	5/0.44	10/0.58	6/0.47	2/0.36	-2/0.25
$S^1{}_3 = 3$	5/0.44	10/0.58	15/0.72	11/0.61	7/0.50
$S^1{}_4 = 4$	5/0.44	10/0.58	15/0.72	20/0.86	16/0.75
$S^1{}_5 = 5$	5/0.44	10/0.58	15/0.72	20/0.86	25/1.00

Source: prepared by the author.

TABLE 2.

Payoff matrix and normalized values (2nd criterion)

Crit. 2	$A_1 = 1$	$A_2 = 2$	$A_3 = 3$	$A_4 = 4$	$A_5 = 5$
$S^2{}_1$	0.5/1.00	0.6/0.96	0.7/0.92	0.8/0.87	0.9/0.83
$S^2{}_2$	1/0.79	1.1/0.75	1.2/0.71	1.3/0.67	1.4/0.62
$S^2{}_3$	2/0.37	2.2/0.29	2.5/0.17	2.7/0.08	2.9/0.00

Source: prepared by the author.

First (step 1), we assume that the DM is a moderate optimist (β=0.7, a=0.3) and that w^1=0.7, w^2=0.3. We normalize values (step 2) since they are expressed in different scales, see Tables 1-2 (second value in each cell). The sums of cumulative relative profits are given in Tables 3-4 (step 3). For example, $p(r)^1{}_{23}$=5· 0.47-(0.22+0.47+0.72+0.72+0.72)=-0.50 and $p(r)^1{}_2$=0.00+0.25-0.5–1.00-1.25=-2.50. Now, we are going to select scenarios with the biggest subjective chance of occurrence (step 4). Measures $p(r)^k{}_\beta$ calculated for each criterion separately show the level of the sum of cumulative relative profits corresponding to the level of the DM's optimism: $p(r)^1{}_\beta$=0.7×(5.00-(-7.50))+(-7.50)=1.25, $p(r)^2{}_\beta$=0.7×(4.71-(-6.29))+(-6.29)=1.41. In the case of the 1st criterion, index $p(r)^1{}_3$ (i.e. for scenario $S^1{}_3$) is exactly equal to $p(r)^1{}_\beta$. Hence, $SS^{1*}=\{S^1{}_3\}$. In the case of the 2nd criterion, there is no scenario for which index $p(r)^2{}_i=p(r)^2{}_\beta$. Thus, according to step 4b, $SS^{2*}=SS^{2*}{}_{\min}\cup SS^{2*}{}_{\max}=\{S^2{}_3\}\cup\{S^2{}_2\}=\{S^2{}_2,S^2{}_3\}$. Within step 5, we compute, for each decision, average values for criterion C_1 (Equation 11): $v^1{}_1$=0.44, $v^1{}_2$=0.58, $v^1{}_3$=0.72, $v^1{}_4$=0.61, $v^1{}_5$=0.50 where m_{1*}=1, and weighted values for criterion C_2 (Equation 12): $v^2{}_1$=0.78, $v^2{}_2$=0.74, $v^2{}_3$=0.70, $v^2{}_4$=0.65, $v^2{}_5$=0.61, where $m^{max}{}_{2*}$=1, $m^{min}{}_{2*}$=1 and $p(r)^{2*}{}_{max}$=1.58, $p(r)^{2*}{}_{min}$=-6.29. In step 6, SAW indices are calculated: SAW_1=0.7·0.44+0.3·0.78=0.55, SAW_2=0.63, SAW_3=0.71, SAW_4=0.62, SAW_5=0.53. According to Equation 14 (step 7), alternative A_3 should be selected, but first, let us check whether for the most important criterion (i.e. the first one: $w^1>w^2$) an appropriate number of normalized outcomes (i.e. at least $z_1 = \lceil (1-0.7)\cdot 5 \rceil = \lceil 0.3\cdot 5 \rceil = 2$) connected with A_3 exceeds the Wald index: $wald^1{}_{best}$=max{0.44, 0.33, 0.22, 0.11, 0.00}=0.44. We see in Table 1 that there are four normalized values equal to at least 0.44, i.e. 0.47, 0.72, 0.72 and 0.72. Therefore, A_3 satisfies condition (15) and becomes the final optimal solution. The newsvendor should buy 3 boxes. This is the optimal pure strategy for one period. If he intends to define the best solution for further periods, he should update his preferences and possibly payoff matrices.

TABLE 3.

Sums of cumulative relative profits (1st criterion)

Crit. 1	$A_1 = 1$	$A_2 = 2$	$A_3 = 3$	$A_4 = 4$	$A_5 = 5$	Sum $p(r)^k{}_i$
$S^1{}_1 = 1$	0.00	-1.00	-1.75	-2.25	-2.50	-7.50
$S^1{}_2 = 2$	0.00	0.25	-0.50	-1.00	-1.25	-2.50
$S^1{}_3 = 3$	0.00	0.25	0.75	0.25	0.00	1.25
$S^1{}_4 = 4$	0.00	0.25	0.75	1.50	1.25	3.75
$S^1{}_5 = 5$	0.00	0.25	0.75	1.50	2.50	5.00

Source: prepared by the author.

TABLE 4.

Sums of cumulative relative profits (2nd criterion)

Crit. 2	$A_1 = 1$	$A_2 = 2$	$A_3 = 3$	$A_4 = 4$	$A_5 = 5$	Sum
$S^2{}_1 = 1$	0.83	0.88	0.96	1.00	1.04	4.71
$S^2{}_2 = 2$	0.21	0.25	0.33	0.38	0.42	1.58
$S^2{}_3 = 3$	-1.04	-1.13	-1.29	-1.38	-1.46	-6.29

Source: prepared by the author.

5. Conclusions

The paper contains a description of a decision rule supporting multi-criteria decision making under uncertainty with unknown probabilities (understood as frequencies). Probabilities are not given here as initial data. Instead of it, some secondary probability-like quantities are used. The goal of the procedure is to find an optimal pure strategy which constitutes a one-shot decision (it is executed only once). The method is designed for games against nature. It is based on two-stage models. Advantages of applying that approach are as follows: 1) It does not require any information about probabilities, which is especially desirable in the case of new decision problems, 2) It takes into consideration the decision-maker's preference structure and nature, but only criteria weights and the level of optimism are supposed to be declared – hence, the procedure can be successfully applied by passive decision-makers, 3) It can be used in the case of totally independent payoff matrices for particular targets. The research takes into account experiment results obtained by [Kahneman 2011] and related to fast and heuristic thinking. The novel rule has been demonstrated by means of an illustrative example concerning the scenario-based bi-criteria single-period newsvendor problem.

In the future it would be desirable to explore the uncertain multi-criteria pure decision-making problem on the assumption that payoffs connected with particular decisions are presented as sets (not sequences) of outcomes, since in some real problems, payoffs connected with particular investments depend on totally different scenarios (even within the framework of a given criterion).

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