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Preface

We are proud to present the special issue of "Formalized Mathematics (a computer assisted approach)" celebrating twenty-five years of the Mizar Mathematical Library (MML).

The 1st of January, 1989 was the official date of acceptance of the first regular Mizar article entitled "Boolean Properties of Sets" (MML identifier: `BOOLE`) by Halina Święczkowska and Zinaida Trybulec. However, earlier approaches to collecting Mizar articles in a coherent form were dated back to the late 1980s. In the very first years of the development of MML main activities were focused on formulating and proving simple properties of basic mathematical objects: families of sets, relations and functions, numbers, etc. Since MML was based on Tarski-Grothendieck set theory, which is very similar to Zermelo-Fraenkel set theory widely used by mathematicians, the article "Tarski Grothendieck Set Theory" containing axioms of set theory, implemented by Andrzej Trybulec, was also an indispensable part of the library. Andrzej Trybulec had been the head of the Mizar project since 1973, hence it's hard to overestimate his influence also on the repository of texts written in the Mizar language.

As of now, MML contains over 1200 articles, over 10 thousand definitions, and approximately 50 thousand theorems. This significant collection has been written by 250 authors.

MML is roughly divided into five parts:

- the axiomatics – currently containing three files with MML identifiers `HIDDEN` (introducing primitive notions), `TARSKI_0` and `TARSKI_A` – basically axioms of Tarski-Grothendieck set theory (actually Tarski's axiom A is the only exportable item in `TARSKI_A`);
- classical part, currently 323 items, not using the notion of a structure – pure set-theoretic part;
- the formal model of random access Turing machines, started by Andrzej Trybulec and Yatsuka Nakamura in 1992, continued until Trybulec's death in 2013;
- structural part – all the other articles; this part deals with the notion of a structure, e.g. algebraic structures such as groups, fields, vector spaces, lattices, etc.;
- Encyclopedia of Mathematics in Mizar (EMM) – currently 14 files with MML identifiers starting with `X`; a collection of monographs.

Extracting MML's classical part is still an ongoing process as some "classical" items are still being formalized. The process of such changes of the library, called *library revisions*, is coordinated by the Library Committee of the Association of Mizar Users. For nearly fifteen years Adam Grabowski has been the head of this committee taking care of the MML as a whole (other persons previously involved in this activity were Edmund Woronowicz and Czesław Byliński).

The printing process of the journal of "Formalized Mathematics" reflected the rapid expansion of MML in its early years. It has stabilized now and is to be published quarterly – with thirty as the approximate number of Mizar articles per volume. Through the years, when the Mizar project evolved, the development of the repository of Mizar texts (including the Mizar language itself) was stimulated by large formalization projects. Among them, the most notable one was the formalization of *Compendium of Continuous Lattices* by Gierz et al. (mentioned in the second edition of the book issued under the title *Continuous Lattices and Domains*) in years 1995–2003; this collective work of over a dozen of Mizar authors resulted in 36 articles from `WAYBEL` series reflecting faithfully the content of the book and 22 articles from `YELLOW` series bridging the gap between the existing and desired state of the MML.

Another example of long-lasting cooperative work of a bigger group of authors was the formalization of the proof of the Jordan Curve Theorem continued from the very beginnings of MML until its successful finale – Artur Korniłowicz's "Jordan Curve Theorem", MML identifier `JORDAN`, accepted on 15th of September, 2005. This may be considered as a part of a more general project: formal encoding of general topology – also influencial throughout the years. Among recently growing parts of mathematics represented in the MML we can list also functional analysis and lattice theory.

The challenge which is stimulating not only for the Mizar system, but also for other proof-assistants is the "Top 100 mathematical theorems" – the collection of important or interesting facts proposed at the edge of centuries by Paul and Jack Abad as "The Hundred Greatest Theorems". On the page maintained by Freek Wiedijk `http://www.cs.ru.nl/F.Wiedijk/100/` one can find systems of computer formalization of mathematics ordered by the number of the items from that list which have been proven in these systems' libraries, covering 90% of items alltogether. Currently, among nine systems listed on the Wiedijk's page, the Mizar system comes in second place with the total number of 62 items formalized:

- HOL Light,
- Mizar,
- Isabelle,
- Coq,
- ProofPower,

- Metamath,
- nqthm/ACL2,
- PVS,
- NuPRL/MetaPRL.

Among the papers published in this special issue, there are formalizations of four problems from this "Top 100" list. Below we quote the formulation of these problems in their original Mizar language form, the theorem number and the page containing its translation into human-readable language, a title, and the name of the author of the corresponding Mizar article.

#19 Four Squares Theorem: page 109, "Lagrange's Four-Square Theorem" by Yasushige Watase, theorem (18):

```
theorem :: LAGRA4SQ:18
  for n be Nat holds
    ex x1,x2,x3,x4 be Nat st n = x1^2 + x2^2 + x3^2 + x4^2;
```

#30 The Ballot Problem: page 122, "Bertrand's Ballot Theorem" by Karol Pąk, theorem (28):

```
theorem :: BALLOT_1:28
  A <> B & n >= k implies
    prob DominatedElection(A,n,B,k) = (n-k) / (n+k);
```

#38 Arithmetic Mean/Geometric Mean: page 165, "Cauchy Mean Theorem" by Adam Grabowski, theorem (47):

```
theorem :: RVSUM_3:47
  for f being non empty positive real-valued FinSequence holds
    GMean f <= Mean f;
```

#54 Königsberg Bridges Problem: page 178, "A Note on the Seven Bridges of Königsberg Problem" by Adam Naumowicz, theorem (5):

```
theorem :: GRAPH_3A:5
  not ex p being Path of KoenigsbergBridges st
    p is cyclic Eulerian;
```

The present issue of "Formalized Mathematics" contains nine articles carefully chosen among various submissions. Every submission is reviewed by at least two (usually three) independent specialists; the reviews are on the double-blind basis. Of course, the same rules apply to all papers contained in this special issue.

Marco B. Caminati with Artur Korniłowicz in "Pseudo-canonical formulae are classical" (MML identifier: `HILBERT4`) give the solution to one of the problems stated by Andrzej Trybulec, being a continuation of his work expressed in the `HILBERT` article series on the Hilbert positive propositional calculus.

Yasushige Watase in "Lagrange's Four-Square Theorem" (LAGRA4SQ) formalized an important well-known fact from basic number theory, suprisingly absent in the Mizar library. The same area of mathematics was developed by Christoph Schwarzweller in "Proth numbers" – (NAT_6) expressing facts which were partially covered in MML in another formulation.

"Bertrand's ballot theorem" by Karol Pąk (BALLOT_1) contains the solution of the famous combinatorial problem connected with the voting process. Grzegorz Bancerek – who is also responsible both for the software translating Mizar texts into the form which can be observed in this volume, as well as the MML Query – the service which enhances searching in the Mizar database, submitted MSAFREE5 – "Term context", continuing his formal description of the Mizar language in terms of universal many-sorted algebras.

"Cauchy mean theorem" (RVSUM_3) authored by Adam Grabowski contains a formal proof of the fact that the arithmetic mean of a list of non-negative real numbers is greater than or equal to the geometric mean of the same list, known briefly as AM–GM inequality. William Richter, Adam Grabowski, and Jesse Alama in their "Tarski Geometry Axioms" (MML Id: GTARSKI1) formalized axioms of geometry proposed by Tarski (it's interesting to observe the fact that the first author formalized them also in HOL Light, in parallel).

In "A note on the seven bridges of Königsberg problem" (GRAPH_3A) Adam Naumowicz gives a brief exposition of the problem of bridges of Königsberg solved originally by Euler – extending graph theory formalized in Mizar. Karol Pąk, in his another submission – "Topological manifolds" (MFOLD_0), continues the series resulting previously in the proof of the Jordan Curve Theorem – algebraic topology.

The publication of this special issue of the journal coincided with the Sixth Podlasie Conference on Mathematics, Białystok, Poland, July 1–4th, 2014, especially with the special session "Computer-assisted Formalization of Mathematics. Andrzej Trybulec in memoriam" organized by Artur Korniłowicz and coorganized by the Faculty of Mathematics and Informatics of the University of Białystok with the enormous support of Professor Krystyna Kuperberg. With this volume, we want to commemorate our late friends – the leaders of the Mizar project, Professors Andrzej Trybulec and Piotr Rudnicki.

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