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A Note on the Seven Bridges of Königsberg Problem

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Summary. In this paper we account for the formalization of the seven bridges of Königsberg puzzle. The problem originally posed and solved by Euler in 1735 is historically notable for having laid the foundations of graph theory, cf. [7]. Our formalization utilizes a simple set-theoretical graph representation with four distinct sets for the graph's vertices and another seven sets that represent the edges (bridges). The work appends the article by Nakamura and Rudnicki [10] by introducing the classic example of a graph that does not contain an Eulerian path.

This theorem is item #54 from the "Formalizing 100 Theorems" list maintained by Freek Wiedijk at http://www.cs.ru.nl/F.Wiedijk/100/ .

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The notation and terminology used in this paper have been introduced in the following articles: [11], [2], [8], [3], [4], [9], [10], [6], [1], [13], [12], and [5].

The functors: KVertices and KEdges yielding sets are defined by terms,

(Def. 1) $\{0, 1, 2, 3\}$,

(Def. 2) $\{10, 20, 30, 40, 50, 60, 70\}$,

respectively. The functors: KSource and KTarget yielding functions from KEdges into KVertices are defined by terms,

(Def. 3) $\{\langle 10, 0\rangle, \langle 20, 0\rangle, \langle 30, 0\rangle, \langle 40, 1\rangle, \langle 50, 1\rangle, \langle 60, 2\rangle, \langle 70, 2\rangle\}$,

(Def. 4) $\{\langle 10, 1\rangle, \langle 20, 2\rangle, \langle 30, 3\rangle, \langle 40, 2\rangle, \langle 50, 2\rangle, \langle 60, 3\rangle, \langle 70, 3\rangle\}$,

respectively. The functor KönigsbergBridges yielding a graph is defined by the term

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(Def. 5) $\langle$KVertices, KEdges, KSource, KTarget$\rangle$.

Let us observe that KönigsbergBridges is finite and connected.

Let us consider a vertex v of KönigsbergBridges. Now we state the propositions:

(1) If $v = 0$, then the degree of $v = 3$. PROOF: EdgesIn $v = \emptyset$ by [3, (1)]. EdgesOut $v = \{10, 20, 30\}$ by [3, (1)]. The degree of $v = 3$ by [10, (24)]. □

(2) If $v = 1$, then the degree of $v = 3$. PROOF: EdgesIn $v = \{10\}$ by [3, (1)]. EdgesOut $v = \{40, 50\}$ by [3, (1)]. The degree of $v = 3$ by [10, (24)]. □

(3) If $v = 2$, then the degree of $v = 5$. PROOF: EdgesIn $v = \{20, 40, 50\}$ by [3, (1)]. EdgesOut $v = \{60, 70\}$ by [3, (1)]. The degree of $v = 5$ by [10, (24)]. □

(4) If $v = 3$, then the degree of $v = 3$. PROOF: EdgesIn $v = \{30, 60, 70\}$ by [3, (1)]. EdgesOut $v = \emptyset$ by [3, (1)]. The degree of $v = 3$ by [10, (24)]. □

Now we state the propositions:

(5) SEVEN BRIDGES OF KÖNIGSBERG:
There exists no path p of KönigsbergBridges such that p is cyclic and Eulerian. The theorem is a consequence of (1).

(6) There exists no path p of KönigsbergBridges such that p is non cyclic and Eulerian. The theorem is a consequence of (4), (1), and (2).

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