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Products in Categories without Uniqueness of cod and dom[1]

Artur Korniłowicz
Institute of Informatics
University of Białystok
Sosnowa 64, 15-887 Białystok
Poland

Summary. The paper introduces Cartesian products in categories without uniqueness of **cod** and **dom**. It is proven that set-theoretical product is the product in the category Ens [7].

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The papers [10], [6], [1], [8], [2], [3], [4], [9], [12], [11], and [5] provide the terminology and notation for this paper.

In this paper I denotes a set and E denotes a non empty set.

Let us mention that every binary relation which is empty is also $\emptyset$-defined.

Let C be a graph. We say that C is functional if and only if:

(Def. 1) For all objects a, b of C holds $\langle a, b \rangle$ is functional.

Let us consider E. One can verify that Ens_E is functional.

Let us observe that there exists a category which is functional and strict.

Let C be a functional category structure. One can verify that the graph of C is functional.

Let us observe that there exists a graph which is functional and strict.

Let us note that there exists a category which is functional and strict.

Let C be a functional graph and let a, b be objects of C. Observe that $\langle a, b \rangle$ is functional.

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Let C be a non empty category structure and let I be a set. An objects family of I and C is a function from I into C.

Let C be a non empty category structure, let o be an object of C, let I be a set, and let f be an objects family of I and C. A many sorted set indexed by I is said to be a morphisms family of o and f if:

(Def. 2) For every set i such that $i \in I$ there exists an object o_1 of C such that $o_1 = f(i)$ and $\mathrm{it}(i)$ is a morphism from o to o_1.

Let C be a non empty category structure, let o be an object of C, let I be a non empty set, and let f be an objects family of I and C. Let us note that the morphisms family of o and f can be characterized by the following (equivalent) condition:

(Def. 3) For every element i of I holds $\mathrm{it}(i)$ is a morphism from o to $f(i)$.

Let C be a non empty category structure, let o be an object of C, let I be a non empty set, let f be an objects family of I and C, let M be a morphisms family of o and f, and let i be an element of I. Then $M(i)$ is a morphism from o to $f(i)$.

Let C be a functional non empty category structure, let o be an object of C, let I be a set, and let f be an objects family of I and C. Observe that every morphisms family of o and f is function yielding.

Next we state the proposition

(1) Let C be a non empty category structure, o be an object of C, and f be an objects family of $\emptyset$ and C. Then $\emptyset$ is a morphisms family of o and f.

Let C be a non empty category structure, let I be a set, let A be an objects family of I and C, let B be an object of C, and let P be a morphisms family of B and A. We say that P is feasible if and only if:

(Def. 4) For every set i such that $i \in I$ there exists an object o of C such that $o = A(i)$ and $P(i) \in \langle B, o \rangle$.

Let C be a non empty category structure, let I be a non empty set, let A be an objects family of I and C, let B be an object of C, and let P be a morphisms family of B and A. Let us observe that P is feasible if and only if:

(Def. 5) For every element i of I holds $P(i) \in \langle B, A(i) \rangle$.

Let C be a category, let I be a set, let A be an objects family of I and C, let B be an object of C, and let P be a morphisms family of B and A. We say that P is projection morphisms family if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let X be an object of C and F be a morphisms family of X and A. Suppose F is feasible. Then there exists a morphism f from X to B such that

(i) $f \in \langle X, B \rangle$,

(ii) for every set i such that $i \in I$ there exists an object s_1 of C and there exists a morphism P_1 from B to s_1 such that $s_1 = A(i)$ and $P_1 = P(i)$ and $F(i) = P_1 \cdot f$, and

(iii) for every morphism f_1 from X to B such that for every set i such that $i \in I$ there exists an object s_1 of C and there exists a morphism P_1 from B to s_1 such that $s_1 = A(i)$ and $P_1 = P(i)$ and $F(i) = P_1 \cdot f_1$ holds $f = f_1$.

Let C be a category, let I be a non empty set, let A be an objects family of I and C, let B be an object of C, and let P be a morphisms family of B and A. Let us observe that P is projection morphisms family if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let X be an object of C and F be a morphisms family of X and A. Suppose F is feasible. Then there exists a morphism f from X to B such that

(i) $f \in \langle X, B \rangle$,

(ii) for every element i of I holds $F(i) = P(i) \cdot f$, and

(iii) for every morphism f_1 from X to B such that for every element i of I holds $F(i) = P(i) \cdot f_1$ holds $f = f_1$.

Let C be a category, let A be an objects family of $\emptyset$ and C, and let B be an object of C. Note that every morphisms family of B and A is feasible.

One can prove the following propositions:

(2) Let C be a category, A be an objects family of $\emptyset$ and C, and B be an object of C. If B is terminal, then there exists a morphisms family of B and A which is empty and projection morphisms family.

(3) For every objects family A of I and Ens_1 and for every object o of Ens_1 holds $I \longmapsto \emptyset$ is a morphisms family of o and A.

(4) Let A be an objects family of I and Ens_1, o be an object of Ens_1, and P be a morphisms family of o and A. If $P = I \longmapsto \emptyset$, then P is feasible and projection morphisms family.

Let C be a category. We say that C has products if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let I be a set and A be an objects family of I and C. Then there exists an object B of C such that there exists a morphisms family of B and A which is feasible and projection morphisms family.

Let us note that Ens_1 has products.

One can check that there exists a category which has products.

Let C be a category, let I be a set, let A be an objects family of I and C, and let B be an object of C. We say that B is A-cat product-like if and only if:

(Def. 9) There exists a morphisms family of B and A which is feasible and projection morphisms family.

Let C be a category with products, let I be a set, and let A be an objects family of I and C. One can check that there exists an object of C which is A-cat product-like.

Let C be a category and let A be an objects family of $\emptyset$ and C. Note that every object of C which is A-cat product-like is also terminal.

We now state two propositions:

(5) Let C be a category, A be an objects family of $\emptyset$ and C, and B be an object of C. If B is terminal, then B is A-cat product-like.

(6) Let C be a category, A be an objects family of I and C, and C_1, C_2 be objects of C. Suppose C_1 is A-cat product-like and C_2 is A-cat product-like. Then C_1, C_2 are iso.

In the sequel A is an objects family of I and Ens_E.

Let us consider I, E, A. Let us assume that $\prod A \in E$. The functor EnsCatProductObj A yielding an object of Ens_E is defined by:

(Def. 10) $\mathrm{EnsCatProductObj}\, A = \prod A$.

Let us consider I, E, A. Let us assume that $\prod A \in E$. The functor EnsCatProduct A yields a morphisms family of EnsCatProductObj A and A and is defined by:

(Def. 11) For every set i such that $i \in I$ holds $(\mathrm{EnsCatProduct}\, A)(i) = \mathrm{proj}(A, i)$.

We now state four propositions:

(7) If $\prod A \in E$ and $\prod A = \emptyset$, then $\mathrm{EnsCatProduct}\, A = I \longmapsto \emptyset$.

(8) If $\prod A \in E$, then EnsCatProduct A is feasible and projection morphisms family.

(9) If $\prod A \in E$, then EnsCatProductObj A is A-cat product-like.

(10) If for all I, A holds $\prod A \in E$, then Ens_E has products.

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