

Integrability Formulas. Part II

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Summary. In this article, we give several differentiation and integrability formulas of special and composite functions including trigonometric function, and polynomial function.

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The terminology and notation used here have been introduced in the following articles: [12], [13], [2], [3], [9], [1], [6], [11], [14], [4], [18], [7], [8], [5], [19], [10], [16], [17], and [15].

For simplicity, we use the following convention: a, x are real numbers, n is an element of \mathbb{N} , A is a closed-interval subset of \mathbb{R} , f, h, f_1, f_2 are partial functions from \mathbb{R} to \mathbb{R} , and Z is an open subset of \mathbb{R} .

The following propositions are true:

- (1) Suppose that
 - (i) $A \subseteq Z$,
 - (ii) $f = \frac{1}{(\text{the function sin})(\text{the function cos})}$,
 - (iii) $Z \subseteq \text{dom}((\text{the function ln}) \cdot (\text{the function tan}))$,
 - (iv) $Z = \text{dom } f$, and
 - (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function ln}) \cdot (\text{the function tan}))(\text{sup } A) - ((\text{the function ln}) \cdot (\text{the function tan}))(\text{inf } A)$.

(2) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = -\frac{1}{(\text{the function sin})(\text{the function cos})}$,
- (iii) $Z \subseteq \text{dom}((\text{the function ln}) \cdot (\text{the function cot}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function ln}) \cdot (\text{the function cot}))(\sup A) - ((\text{the function ln}) \cdot (\text{the function cot}))(\inf A)$.

(3) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = 2((\text{the function exp})(\text{the function sin}))$,
- (iii) $Z \subseteq \text{dom}((\text{the function exp})((\text{the function sin}) - (\text{the function cos})))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function exp})((\text{the function sin}) - (\text{the function cos}))) (\sup A) - ((\text{the function exp})((\text{the function sin}) - (\text{the function cos}))) (\inf A)$.

(4) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = 2((\text{the function exp})(\text{the function cos}))$,
- (iii) $Z \subseteq \text{dom}((\text{the function exp})((\text{the function sin}) + (\text{the function cos})))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function exp})((\text{the function sin}) + (\text{the function cos}))) (\sup A) - ((\text{the function exp})((\text{the function sin}) + (\text{the function cos}))) (\inf A)$.

(5) Suppose $A \subseteq Z = \text{dom}((\text{the function cos}) - (\text{the function sin}))$ and $(\text{the function cos}) - (\text{the function sin})$ is continuous on A .

Then $\int_A ((\text{the function cos}) - (\text{the function sin}))(x)dx = ((\text{the function sin}) + (\text{the function cos}))(\sup A) - ((\text{the function sin}) + (\text{the function cos}))(\inf A)$.

(6) Suppose $A \subseteq Z = \text{dom}((\text{the function cos}) + (\text{the function sin}))$ and $(\text{the function cos}) + (\text{the function sin})$ is continuous on A .

Then $\int_A ((\text{the function cos}) + (\text{the function sin}))(x)dx = ((\text{the function sin}) - (\text{the function cos}))(\sup A) - ((\text{the function sin}) - (\text{the function cos}))(\inf A)$.

- (7) Suppose $Z \subseteq \text{dom}((-\frac{1}{2}) \frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})$. Then
- (i) $(-\frac{1}{2}) \frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds

$$((-\frac{1}{2}) \frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})'_{|Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function exp})(x)}.$$

(8) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{\text{the function sin}}{\text{the function exp}}$,
- (iii) $Z \subseteq \text{dom}((-\frac{1}{2}) \frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

$$\text{Then } \int_A f(x)dx = ((-\frac{1}{2}) \frac{(\text{the function sin}) + (\text{the function cos})}{\text{the function exp}})(\text{sup } A) - ((-\frac{1}{2}) \frac{(\text{the function sin}) + (\text{the function cos})}{\text{the function exp}})(\text{inf } A).$$

(9) Suppose $Z \subseteq \text{dom}(\frac{1}{2} \frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})$. Then

- (i) $\frac{1}{2} \frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds

$$(\frac{1}{2} \frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})'_{|Z}(x) = \frac{(\text{the function cos})(x)}{(\text{the function exp})(x)}.$$

(10) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{\text{the function cos}}{\text{the function exp}}$,
- (iii) $Z \subseteq \text{dom}(\frac{1}{2} \frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

$$\text{Then } \int_A f(x)dx = (\frac{1}{2} \frac{(\text{the function sin}) - (\text{the function cos})}{\text{the function exp}})(\text{sup } A) - (\frac{1}{2} \frac{(\text{the function sin}) - (\text{the function cos})}{\text{the function exp}})(\text{inf } A).$$

(11) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = (\text{the function exp}) ((\text{the function sin})+(\text{the function cos}))$,
- (iii) $Z \subseteq \text{dom}((\text{the function exp}) (\text{the function sin}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

$$\text{Then } \int_A f(x)dx = ((\text{the function exp}) (\text{the function sin}))(\text{sup } A) - ((\text{the function exp}) (\text{the function sin}))(\text{inf } A).$$

(12) Suppose that

- (i) $A \subseteq Z$,

- (ii) $f = (\text{the function exp}) ((\text{the function cos}) - (\text{the function sin}))$,
- (iii) $Z \subseteq \text{dom}((\text{the function exp}) (\text{the function cos}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function exp}) (\text{the function cos}))(\text{sup } A) - ((\text{the function exp}) (\text{the function cos}))(\text{inf } A)$.

(13) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f_1 = \square^2$,
- (iii) $f = -\frac{\frac{\text{the function sin}}{\text{the function cos}}}{f_1} + \frac{\frac{1}{\text{id}_Z}}{(\text{the function cos})^2}$,
- (iv) $Z \subseteq \text{dom}(\frac{1}{\text{id}_Z} (\text{the function tan}))$,
- (v) $Z = \text{dom } f$, and
- (vi) f is continuous on A .

Then $\int_A f(x)dx = (\frac{1}{\text{id}_Z} (\text{the function tan}))(\text{sup } A) - (\frac{1}{\text{id}_Z} (\text{the function tan}))(\text{inf } A)$.

(14) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = -\frac{\frac{\text{the function cos}}{\text{the function sin}}}{f_1} - \frac{\frac{1}{\text{id}_Z}}{(\text{the function sin})^2}$,
- (iii) $f_1 = \square^2$,
- (iv) $Z \subseteq \text{dom}(\frac{1}{\text{id}_Z} (\text{the function cot}))$,
- (v) $Z = \text{dom } f$, and
- (vi) f is continuous on A .

Then $\int_A f(x)dx = (\frac{1}{\text{id}_Z} (\text{the function cot}))(\text{sup } A) - (\frac{1}{\text{id}_Z} (\text{the function cot}))(\text{inf } A)$.

(15) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{\frac{\text{the function sin}}{\text{the function cos}}}{\text{id}_Z} + \frac{\text{the function ln}}{(\text{the function cos})^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function ln}) (\text{the function tan}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function ln}) (\text{the function tan}))(\text{sup } A) - ((\text{the function ln}) (\text{the function tan}))(\text{inf } A)$.

(16) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{\frac{\text{the function cos}}{\text{the function sin}}}{\text{id}_Z} - \frac{\text{the function ln}}{(\text{the function sin})^2}$,

- (iii) $Z \subseteq \text{dom}(\text{the function } \ln \text{ (the function } \cot)),$
- (iv) $Z = \text{dom } f,$ and
- (v) f is continuous on $A.$

Then $\int_A f(x)dx = ((\text{the function } \ln \text{ (the function } \cot))(\sup A) - ((\text{the function } \ln \text{ (the function } \cot))(\inf A)).$

(17) Suppose that

- (i) $A \subseteq Z,$
- (ii) $f = \frac{\text{the function } \tan}{\text{id}_Z} + \frac{\text{the function } \ln}{(\text{the function } \cos)^2},$
- (iii) $Z \subseteq \text{dom}(\text{the function } \ln \text{ (the function } \tan)),$
- (iv) $Z \subseteq \text{dom}(\text{the function } \tan),$
- (v) $Z = \text{dom } f,$ and
- (vi) f is continuous on $A.$

Then $\int_A f(x)dx = ((\text{the function } \ln \text{ (the function } \tan))(\sup A) - ((\text{the function } \ln \text{ (the function } \tan))(\inf A)).$

(18) Suppose that

- (i) $A \subseteq Z,$
- (ii) $f = \frac{\text{the function } \cot}{\text{id}_Z} - \frac{\text{the function } \ln}{(\text{the function } \sin)^2},$
- (iii) $Z \subseteq \text{dom}(\text{the function } \ln \text{ (the function } \cot)),$
- (iv) $Z \subseteq \text{dom}(\text{the function } \cot),$
- (v) $Z = \text{dom } f,$ and
- (vi) f is continuous on $A.$

Then $\int_A f(x)dx = ((\text{the function } \ln \text{ (the function } \cot))(\sup A) - ((\text{the function } \ln \text{ (the function } \cot))(\inf A)).$

(19) Suppose that

- (i) $A \subseteq Z,$
- (ii) for every x such that $x \in Z$ holds $f_1(x) = 1,$
- (iii) $f = \frac{\text{the function } \arctan}{\text{id}_Z} + \frac{\text{the function } \ln}{f_1 + \square^2},$
- (iv) $Z \subseteq]-1, 1[,$
- (v) $Z = \text{dom } f,$ and
- (vi) f is continuous on $A.$

Then $\int_A f(x)dx = ((\text{the function } \ln \text{ (the function } \arctan))(\sup A) - ((\text{the function } \ln \text{ (the function } \arctan))(\inf A)).$

(20) Suppose that

- (i) $A \subseteq Z,$
- (ii) for every x such that $x \in Z$ holds $f_1(x) = 1,$
- (iii) $f = \frac{\text{the function } \text{arccot}}{\text{id}_Z} - \frac{\text{the function } \ln}{f_1 + \square^2},$

- (iv) $Z \subseteq]-1, 1[$,
- (v) $Z = \text{dom } f$, and
- (vi) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function ln}) \cdot (\text{the function arccot}))(\text{sup } A) - ((\text{the function ln}) \cdot (\text{the function arccot}))(\text{inf } A)$.

- (21) Suppose $A \subseteq Z$ and $f = \frac{(\text{the function exp}) \cdot (\text{the function tan})}{(\text{the function cos})^2}$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = ((\text{the function exp}) \cdot (\text{the function tan}))(\text{sup } A) - ((\text{the function exp}) \cdot (\text{the function tan}))(\text{inf } A)$.

- (22) Suppose $A \subseteq Z$ and $f = -\frac{(\text{the function exp}) \cdot (\text{the function cot})}{(\text{the function sin})^2}$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = ((\text{the function exp}) \cdot (\text{the function cot}))(\text{sup } A) - ((\text{the function exp}) \cdot (\text{the function cot}))(\text{inf } A)$.

- (23) Suppose $Z \subseteq \text{dom}((\text{the function exp}) \cdot (\text{the function cot}))$. Then
- (i) $-(\text{the function exp}) \cdot (\text{the function cot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $-(\text{the function exp}) \cdot (\text{the function cot})|_Z(x) = \frac{(\text{the function exp}) \cdot ((\text{the function cot})(x))}{(\text{the function sin})(x)^2}$.

- (24) Suppose $A \subseteq Z$ and $f = \frac{(\text{the function exp}) \cdot (\text{the function cot})}{(\text{the function sin})^2}$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = (-\text{the function exp}) \cdot (\text{the function cot})(\text{sup } A) - (-\text{the function exp}) \cdot (\text{the function cot})(\text{inf } A)$.

- (25) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{1}{\text{id}_Z((\text{the function cos}) \cdot (\text{the function ln}))^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function tan}) \cdot (\text{the function ln}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function tan}) \cdot (\text{the function ln}))(\text{sup } A) - ((\text{the function tan}) \cdot (\text{the function ln}))(\text{inf } A)$.

- (26) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = -\frac{1}{\text{id}_Z((\text{the function sin}) \cdot (\text{the function ln}))^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function ln}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function cot}) \cdot (\text{the function ln}))(\text{sup } A) - ((\text{the function cot}) \cdot (\text{the function ln}))(\text{inf } A)$.

(27) Suppose $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function ln}))$. Then

- (i) $-(\text{the function cot}) \cdot (\text{the function ln})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(-(\text{the function cot}) \cdot (\text{the function ln}))'_Z(x) = \frac{1}{x \cdot (\text{the function sin})((\text{the function ln})(x))^2}$.

(28) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{1}{\text{id}_Z((\text{the function sin}) \cdot (\text{the function ln}))^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function ln}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = (-\text{the function cot}) \cdot (\text{the function ln})(\text{sup } A) - (-\text{the function cot}) \cdot (\text{the function ln})(\text{inf } A)$.

(29) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = \frac{\text{the function exp}}{((\text{the function cos}) \cdot (\text{the function exp}))^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function tan}) \cdot (\text{the function exp}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function tan}) \cdot (\text{the function exp}))(\text{sup } A) - ((\text{the function tan}) \cdot (\text{the function exp}))(\text{inf } A)$.

(30) Suppose that

- (i) $A \subseteq Z$,
- (ii) $f = -\frac{\text{the function exp}}{((\text{the function sin}) \cdot (\text{the function exp}))^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function exp}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function cot}) \cdot (\text{the function exp}))(\text{sup } A) - ((\text{the function cot}) \cdot (\text{the function exp}))(\text{inf } A)$.

(31) Suppose $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function exp}))$. Then

- (i) $-(\text{the function cot}) \cdot (\text{the function exp})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(-(\text{the function cot}) \cdot (\text{the function exp}))'_Z(x) = \frac{(\text{the function exp})(x)}{(\text{the function sin})((\text{the function exp})(x))^2}$.

(32) Suppose that

- (i) $A \subseteq Z$,

- (ii) $f = \frac{\text{the function exp}}{((\text{the function sin}) \cdot (\text{the function exp}))^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function exp}))$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = (-\text{(the function cot)} \cdot \text{(the function exp)})(\text{sup } A) - (-\text{(the function cot)} \cdot \text{(the function exp)})(\text{inf } A)$.

(33) Suppose that

- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f(x) = -\frac{1}{x^2 \cdot (\text{the function cos})(\frac{1}{x})^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function tan}) \cdot \frac{1}{\text{id}_Z})$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function tan}) \cdot \frac{1}{\text{id}_Z})(\text{sup } A) - ((\text{the function tan}) \cdot \frac{1}{\text{id}_Z})(\text{inf } A)$.

(34) Suppose $Z \subseteq \text{dom}((\text{the function tan}) \cdot \frac{1}{\text{id}_Z})$. Then

- (i) $-\text{(the function tan)} \cdot \frac{1}{\text{id}_Z}$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(-\text{(the function tan)} \cdot \frac{1}{\text{id}_Z})'_Z(x) = \frac{1}{x^2 \cdot (\text{the function cos})(\frac{1}{x})^2}$.

(35) Suppose that

- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f(x) = \frac{1}{x^2 \cdot (\text{the function cos})(\frac{1}{x})^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function tan}) \cdot \frac{1}{\text{id}_Z})$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = (-\text{(the function tan)} \cdot \frac{1}{\text{id}_Z})(\text{sup } A) - (-\text{(the function tan)} \cdot \frac{1}{\text{id}_Z})(\text{inf } A)$.

(36) Suppose that

- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f(x) = \frac{1}{x^2 \cdot (\text{the function sin})(\frac{1}{x})^2}$,
- (iii) $Z \subseteq \text{dom}((\text{the function cot}) \cdot \frac{1}{\text{id}_Z})$,
- (iv) $Z = \text{dom } f$, and
- (v) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function cot}) \cdot \frac{1}{\text{id}_Z})(\text{sup } A) - ((\text{the function cot}) \cdot \frac{1}{\text{id}_Z})(\text{inf } A)$.

- (37) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $(\text{the function arctan})(x) > 0$ and $f = \frac{1}{(f_1 + \square^2) \cdot \text{the function arctan}}$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\text{the function ln}) \cdot (\text{the function arctan}))$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = ((\text{the function ln}) \cdot (\text{the function arctan}))(\text{sup } A) - ((\text{the function ln}) \cdot (\text{the function arctan}))(\text{inf } A)$.
- (38) Suppose that $A \subseteq Z$ and $f = n \frac{(\square^{n-1}) \cdot \text{the function arctan}}{f_1 + \square^2}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\square^n) \cdot \text{the function arctan})$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = ((\square^n) \cdot \text{the function arctan})(\text{sup } A) - ((\square^n) \cdot \text{the function arctan})(\text{inf } A)$.
- (39) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = -n \frac{(\square^{n-1}) \cdot \text{the function arccot}}{f_1 + \square^2}$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\square^n) \cdot \text{the function arccot})$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = ((\square^n) \cdot \text{the function arccot})(\text{sup } A) - ((\square^n) \cdot \text{the function arccot})(\text{inf } A)$.
- (40) Suppose $Z \subseteq \text{dom}((\square^n) \cdot \text{the function arccot})$ and $Z \subseteq]-1, 1[$. Then
- (i) $-(\square^n) \cdot \text{the function arccot}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(-(\square^n) \cdot \text{the function arccot})'|_Z(x) = \frac{n \cdot (\text{the function arccot})(x)^{n-1}}{1+x^2}$.
- (41) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = n \frac{(\square^{n-1}) \cdot \text{the function arccot}}{f_1 + \square^2}$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\square^n) \cdot \text{the function arccot})$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = (-(\square^n) \cdot \text{the function arccot})(\text{sup } A) - (-(\square^n) \cdot \text{the function arccot})(\text{inf } A)$.
- (42) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arctan}}{f_1 + \square^2}$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\square^2) \cdot \text{the function arctan})$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = (\frac{1}{2} ((\square^2) \cdot \text{the function arctan}))(\text{sup } A) - (\frac{1}{2} ((\square^2) \cdot \text{the function arctan}))(\text{inf } A)$.
- (43) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = -\frac{\text{the function arccot}}{f_1 + \square^2}$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\square^2) \cdot \text{the function arccot})$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = (\frac{1}{2} ((\square^2) \cdot \text{the function arccot}))(\text{sup } A) - (\frac{1}{2} ((\square^2) \cdot \text{the function arccot}))(\text{inf } A)$.
- (44) Suppose $Z \subseteq \text{dom}((\square^2) \cdot \text{the function arccot})$ and $Z \subseteq]-1, 1[$. Then
- (i) $-\frac{1}{2} ((\square^2) \cdot \text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds

$$\left(-\frac{1}{2}((\square^2) \cdot \text{the function arccot})\right)'|_Z(x) = \frac{(\text{the function arccot})(x)}{1+x^2}.$$

- (45) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arccot}}{f_1 + \square^2}$ and $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}((\square^2) \cdot \text{the function arccot})$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = \left(-\frac{1}{2}((\square^2) \cdot \text{the function arccot})\right)(\text{sup } A) - \left(-\frac{1}{2}((\square^2) \cdot \text{the function arccot})\right)(\text{inf } A)$.

- (46) Suppose that

- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
- (iii) $f = (\text{the function arctan}) + \frac{\text{id}_Z}{f_1 + \square^2}$,
- (iv) $Z \subseteq]-1, 1[$,
- (v) $Z = \text{dom } f$, and
- (vi) f is continuous on A .

$$\text{Then } \int_A f(x)dx = (\text{id}_Z \text{ the function arctan})(\text{sup } A) - (\text{id}_Z \text{ the function arctan})(\text{inf } A).$$

- (47) Suppose that

- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
- (iii) $f = (\text{the function arccot}) - \frac{\text{id}_Z}{f_1 + \square^2}$,
- (iv) $Z \subseteq]-1, 1[$,
- (v) $Z = \text{dom } f$, and
- (vi) f is continuous on A .

$$\text{Then } \int_A f(x)dx = (\text{id}_Z \text{ the function arccot})(\text{sup } A) - (\text{id}_Z \text{ the function arccot})(\text{inf } A).$$

- (48) Suppose that

- (i) $A \subseteq Z$,
- (ii) $Z \subseteq]-1, 1[$,
- (iii) $f = \frac{(\text{the function exp}) \cdot (\text{the function arctan})}{f_1 + \square^2}$,
- (iv) for every x such that $x \in Z$ holds $f_1(x) = 1$,
- (v) $Z = \text{dom } f$, and
- (vi) f is continuous on A .

$$\text{Then } \int_A f(x)dx = ((\text{the function exp}) \cdot (\text{the function arctan}))(\text{sup } A) - ((\text{the function exp}) \cdot (\text{the function arctan}))(\text{inf } A).$$

- (49) Suppose that

- (i) $A \subseteq Z$,
- (ii) $Z \subseteq]-1, 1[$,

- (iii) $f = -\frac{(\text{the function exp}) \cdot (\text{the function arccot})}{f_1 + \square^2}$,
 (iv) for every x such that $x \in Z$ holds $f_1(x) = 1$,
 (v) $Z = \text{dom } f$, and
 (vi) f is continuous on A .

Then $\int_A f(x)dx = ((\text{the function exp}) \cdot (\text{the function arccot}))(\text{sup } A) - ((\text{the function exp}) \cdot (\text{the function arccot}))(\text{inf } A)$.

- (50) Suppose $Z \subseteq \text{dom}((\text{the function exp}) \cdot (\text{the function arccot}))$ and $Z \subseteq]-1, 1[$. Then

- (i) $-(\text{the function exp}) \cdot (\text{the function arccot})$ is differentiable on Z , and
 (ii) for every x such that $x \in Z$ holds $(-(\text{the function exp}) \cdot (\text{the function arccot}))'_Z(x) = \frac{(\text{the function exp})(\text{the function arccot})(x)}{1+x^2}$.

- (51) Suppose that

- (i) $A \subseteq Z$,
 (ii) $Z \subseteq]-1, 1[$,
 (iii) $f = \frac{(\text{the function exp}) \cdot (\text{the function arccot})}{f_1 + \square^2}$,
 (iv) for every x such that $x \in Z$ holds $f_1(x) = 1$,
 (v) $Z = \text{dom } f$, and
 (vi) f is continuous on A .

Then $\int_A f(x)dx = (-(\text{the function exp}) \cdot (\text{the function arccot}))(\text{sup } A) - (-(\text{the function exp}) \cdot (\text{the function arccot}))(\text{inf } A)$.

- (52) Suppose that $A \subseteq Z \subseteq \text{dom}((\text{the function ln}) \cdot (f_1 + f_2))$ and $f = \frac{\text{id}_Z}{f_1 + f_2}$ and $f_2 = \square^2$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = \frac{1}{2} ((\text{the function ln}) \cdot (f_1 + f_2))(\text{sup } A) - \frac{1}{2} ((\text{the function ln}) \cdot (f_1 + f_2))(\text{inf } A)$.

- (53) Suppose that $A \subseteq Z \subseteq \text{dom}((\text{the function ln}) \cdot (f_1 + f_2))$ and $f = \frac{\text{id}_Z}{a(f_1 + f_2)}$ and for every x such that $x \in Z$ holds $h(x) = \frac{x}{a}$ and $f_1(x) = 1$ and $a \neq 0$ and $f_2 = (\square^2) \cdot h$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x)dx = \left(\frac{a}{2}\right) ((\text{the function ln}) \cdot (f_1 + f_2))(\text{sup } A) - \left(\frac{a}{2}\right) ((\text{the function ln}) \cdot (f_1 + f_2))(\text{inf } A)$.

- (54) Suppose $Z \subseteq \text{dom}\left(\frac{1}{\text{id}_Z} \text{ the function arctan}\right)$ and $Z \subseteq]-1, 1[$. Then

- (i) $-\frac{1}{\text{id}_Z} \text{ the function arctan}$ is differentiable on Z , and
 (ii) for every x such that $x \in Z$ holds $(-\frac{1}{\text{id}_Z} \text{ the function arctan})'_Z(x) = \frac{(\text{the function arctan})(x)}{x^2} - \frac{1}{x \cdot (1+x^2)}$.

- (55) Suppose $Z \subseteq \text{dom}\left(\frac{1}{\text{id}_Z} \text{ the function arccot}\right)$ and $Z \subseteq]-1, 1[$. Then

- (i) $-\frac{1}{\text{id}_Z} \text{ the function arccot}$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $(-\frac{1}{\text{id}_Z} \text{ the function arccot})'_{|Z}(x) = \frac{(\text{the function arccot})(x)}{x^2} + \frac{1}{x \cdot (1+x^2)}$.
- (56) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arctan}}{\square^2} - \frac{1}{\text{id}_Z (f_1 + \square^2)}$ and $Z \subseteq \text{dom}(\frac{1}{\text{id}_Z} \text{ the function arctan})$ and $Z \subseteq]-1, 1[$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x) dx = (-\frac{1}{\text{id}_Z} \text{ the function arctan})(\text{sup } A) - (-\frac{1}{\text{id}_Z} \text{ the function arctan})(\text{inf } A)$.
- (57) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arccot}}{\square^2} + \frac{1}{\text{id}_Z (f_1 + \square^2)}$ and $Z \subseteq \text{dom}(\frac{1}{\text{id}_Z} \text{ the function arccot})$ and $Z \subseteq]-1, 1[$ and $Z = \text{dom } f$ and f is continuous on A . Then $\int_A f(x) dx = (-\frac{1}{\text{id}_Z} \text{ the function arccot})(\text{sup } A) - (-\frac{1}{\text{id}_Z} \text{ the function arccot})(\text{inf } A)$.

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