Integrability Formulas. Part I

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Summary. In this article, we give several differentiation and integrability formulas of special and composite functions including the trigonometric function, and the polynomial function.

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The papers [12], [2], [3], [1], [7], [11], [13], [4], [17], [8], [9], [6], [18], [5], [10], [15], [16], and [14] provide the terminology and notation for this paper.

One can check that there exists a subset of $\mathbb{R}$ which is closed-interval.

For simplicity, we use the following convention: $a$, $b$, $x$, $r$ are real numbers, $n$ is an element of $\mathbb{N}$, $A$ is a closed-interval subset of $\mathbb{R}$, $f$, $g$, $f_1$, $f_2$, $g_1$, $g_2$ are partial functions from $\mathbb{R}$ to $\mathbb{R}$, and $Z$ is an open subset of $\mathbb{R}$.

We now state a number of propositions:

1. Suppose $Z \subseteq \text{dom}(\frac{1}{f_1+f_2})$ and for every $x$ such that $x \in Z$ holds $f_1(x) = 1$ and $f_2 = \square^2$. Then $\frac{1}{f_1+f_2}$ is differentiable on $Z$ and for every $x$ such that $x \in Z$ holds $(\frac{1}{f_1+f_2})'(x) = -\frac{2x}{(1+x^2)^2}$.

2. Suppose that $A \subseteq Z$ and $f = \frac{1}{f_1+f_2}$ and $f_2 = \text{the function } \text{arccot}$ and $Z \subseteq ]-1,1[ \text{ and } g_2 = \square^2$ and for every $x$ such that $x \in Z$ holds $g_1(x) = 1$ and $f_2(x) > 0$ and $Z = \text{dom } f$.

Then $\int_A f(x)dx = (-(\text{the function } \text{ln}) \cdot (\text{the function } \text{arccot}))(\text{sup } A) - (-(\text{the function } \text{ln}) \cdot (\text{the function } \text{arccot}))(\text{inf } A)$.

3. Suppose that

(i) $A \subseteq Z$, 

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(ii) for every \( x \) such that \( x \in Z \) holds \((\text{the function exp})(x) < 1\) and \( f_1(x) = 1 \),
(iii) \( Z \subseteq \text{dom}(\text{the function arctan}) \cdot (\text{the function exp})\),
(iv) \( Z = \text{dom} \ f, \) and
(v) \( f = \frac{\text{the function exp}}{f_1 + (\text{the function exp})^2} \).

Then \( \int_A f(x) \, dx = ((\text{the function arctan}) \cdot (\text{the function exp}))(\sup A) - ((\text{the function arctan}) \cdot (\text{the function exp}))(\inf A) \).

(4) Suppose that
(i) \( A \subseteq Z \),
(ii) for every \( x \) such that \( x \in Z \) holds \((\text{the function exp})(x) < 1\) and \( f_1(x) = 1 \),
(iii) \( Z \subseteq \text{dom}(\text{the function arccot}) \cdot (\text{the function exp})\),
(iv) \( Z = \text{dom} \ f, \) and
(v) \( f = \frac{-\text{the function exp}}{f_1 + (\text{the function exp})^2} \).

Then \( \int_A f(x) \, dx = ((\text{the function arccot}) \cdot (\text{the function exp}))(\sup A) - ((\text{the function arccot}) \cdot (\text{the function exp}))(\inf A) \).

(5) Suppose that
(i) \( A \subseteq Z \),
(ii) \( Z = \text{dom} \ f, \) and
(iii) \( f = (\text{the function exp}) \frac{\text{the function sin}}{\text{the function cos}} + \frac{\text{the function exp}}{(\text{the function cos})^2} \).

Then \( \int_A f(x) \, dx = ((\text{the function exp}) \cdot (\text{the function tan}))(\sup A) - ((\text{the function exp}) \cdot (\text{the function tan}))(\inf A) \).

(6) Suppose that
(i) \( A \subseteq Z \),
(ii) \( Z = \text{dom} \ f, \) and
(iii) \( f = (\text{the function exp}) \frac{-\text{the function cos}}{\text{the function sin}} - \frac{\text{the function exp}}{(\text{the function sin})^2} \).

Then \( \int_A f(x) \, dx = ((\text{the function exp}) \cdot (\text{the function cot}))(\sup A) - ((\text{the function exp}) \cdot (\text{the function cot}))(\inf A) \).

(7) Suppose that
(i) \( A \subseteq Z \),
(ii) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = 1 \),
(iii) \( Z \subseteq \{ -1, 1 \} \),
(iv) \( Z = \text{dom} \ f, \) and
(v) \( f = (\text{the function exp}) \cdot (\text{the function arctan}) + \frac{\text{the function exp}}{f_1 + \text{L}_p} \).
Then $\int_A f(x)\,dx = ((\text{function exp}) \cdot (\text{function arctan}))(\sup A) - ((\text{function exp}) \cdot (\text{function arctan}))(\inf A)$.

(8) Suppose that

(i) $A \subseteq Z$,
(ii) for every $x$ such that $x \in Z$ holds $f_1(x) = 1$, 
(iii) $Z \subseteq ]-1, 1[,$
(iv) $Z = \text{dom } f$, and
(v) $f = (\text{function exp}) \cdot (\text{function arccot}) - \frac{\text{the function exp}}{f_1 + 1^p}.$

Then $\int_A f(x)\,dx = ((\text{function exp}) \cdot (\text{function arccot}))(\sup A) - ((\text{function exp}) \cdot (\text{function arccot}))(\inf A)$.

(9) Suppose $A \subseteq Z = \text{dom } f$ and $f = ((\text{function exp}) \cdot (\text{function sin}) \cdot (\text{function cos})).$ Then $\int_A f(x)\,dx = ((\text{function exp}) \cdot (\text{function sin}))(\sup A) - ((\text{function exp}) \cdot (\text{function sin}))(\inf A)$.

(10) Suppose $A \subseteq Z = \text{dom } f$ and $f = ((\text{function exp}) \cdot (\text{function cos})(\text{function sin})).$ Then $\int_A f(x)\,dx = (-((\text{function exp}) \cdot (\text{function cos}))(\sup A) - ((\text{function exp}) \cdot (\text{function cos}))(\inf A)$.

(11) Suppose $A \subseteq Z$ and for every $x$ such that $x \in Z$ holds $x > 0$ and $Z = \text{dom } f$ and $f = ((\text{function cos}) \cdot (\text{function ln})) \frac{1}{x^p}.$ Then $\int_A f(x)\,dx = ((\text{function sin}) \cdot (\text{function ln}))(\sup A) - ((\text{function sin}) \cdot (\text{function ln}))(\inf A)$.

(12) Suppose $A \subseteq Z$ and for every $x$ such that $x \in Z$ holds $x > 0$ and $Z = \text{dom } f$ and $f = ((\text{function sin}) \cdot (\text{function ln})) \frac{1}{x^p}.$ Then $\int_A f(x)\,dx = (-((\text{function cos}) \cdot (\text{function ln}))(\sup A) - ((\text{function cos}) \cdot (\text{function ln}))(\inf A)$.

(13) Suppose $A \subseteq Z = \text{dom } f$ and $f = (\text{the function exp}) ((\text{function cos}) \cdot (\text{function exp})).$ Then $\int_A f(x)\,dx = ((\text{function sin}) \cdot (\text{function exp}))(\sup A) - ((\text{function sin}) \cdot (\text{function exp}))(\inf A)$.

(14) Suppose $A \subseteq Z = \text{dom } f$ and $f = (\text{the function exp}) ((\text{function sin}) \cdot (\text{function exp})).$ Then $\int_A f(x)\,dx = (-((\text{function cos}) \cdot (\text{function exp}))(\sup A) - ((\text{function cos}) \cdot (\text{function exp}))(\inf A)$.
(15) Suppose that \( A \subseteq Z \subseteq \text{dom}((\text{the function } \ln) \cdot (f_1 + f_2)) \) and \( r \neq 0 \) and for every \( x \) such that \( x \in Z \) holds \( g(x) = \frac{x}{r} \) and \( g(x) > -1 \) and \( g(x) < 1 \) and \( f_1(x) = 1 \) and \( f_2 = (\bigcirc^2) \cdot g \) and \( Z = \text{dom} f \) and \( f = (\text{the function } \arctan) \cdot g \). Then \( \int_A f(x)dx = (\text{id}_Z ((\text{the function } \arctan) \cdot g) - \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))((\sup A) - (\text{id}_Z ((\text{the function } \arctan) \cdot g) - \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))((\inf A)). \)

(16) Suppose that \( A \subseteq Z \subseteq \text{dom}((\text{the function } \ln) \cdot (f_1 + f_2)) \) and \( r \neq 0 \) and for every \( x \) such that \( x \in Z \) holds \( g(x) = \frac{x}{r} \) and \( g(x) > -1 \) and \( g(x) < 1 \) and \( f_1(x) = 1 \) and \( f_2 = (\bigcirc^2) \cdot g \) and \( Z = \text{dom} f \) and \( f = (\text{the function } \arccot) \cdot g \). Then \( \int_A f(x)dx = (\text{id}_Z ((\text{the function } \arccot) \cdot g) + \frac{r}{2} ((\text{the function } \ln) \cdot (f_1 + f_2)))((\inf A)). \)

(17) Suppose that

(i) \( A \subseteq Z \),
(ii) \( f = (\text{the function } \arctan) \cdot f_1 + \frac{\text{id}_x}{r(g+f_1x)} \).
(iii) for every \( x \) such that \( x \in Z \) holds \( g(x) = 1 \) and \( f_1(x) = \frac{x}{r} \) and \( f_1(x) > -1 \) and \( f_1(x) < 1 \),
(iv) \( Z = \text{dom} f \), and
(v) \( f \) is continuous on \( A \).

Then \( \int_A f(x)dx = (\text{id}_Z ((\text{the function } \arctan) \cdot f_1))((\sup A) - (\text{id}_Z ((\text{the function } \arctan) \cdot f_1)))((\inf A)). \)

(18) Suppose that

(i) \( A \subseteq Z \),
(ii) \( f = (\text{the function } \arccot) \cdot f_1 - \frac{\text{id}_x}{r(g+f_1x)} \).
(iii) for every \( x \) such that \( x \in Z \) holds \( g(x) = 1 \) and \( f_1(x) = \frac{x}{r} \) and \( f_1(x) > -1 \) and \( f_1(x) < 1 \),
(iv) \( Z = \text{dom} f \), and
(v) \( f \) is continuous on \( A \).

Then \( \int_A f(x)dx = (\text{id}_Z ((\text{the function } \arccot) \cdot f_1))((\sup A) - (\text{id}_Z ((\text{the function } \arccot) \cdot f_1)))((\inf A)). \)

(19) Suppose that \( A \subseteq Z \subseteq [-1,1] \) and for every \( x \) such that \( x \in Z \) holds \( f_1(x) = 1 \) and \( Z = \text{dom} f \) and \( Z \subseteq \text{dom}((\bigcirc^n) \cdot (\text{the function } \arcsin)) \) and \( 1 < n \) and \( f = \frac{n}{((\bigcirc^{n-1}) \cdot (\text{the function } \arcsin))} \). Then \( \int_A f(x)dx = (((\bigcirc^n) \cdot (\text{the function } \arcsin))((\sup A) - ((\bigcirc^n) \cdot (\text{the function } \arcsin)))((\inf A)). \)

(20) Suppose that \( A \subseteq Z \subseteq [-1,1] \) and for every \( x \) such that \( x \in Z \) holds
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\[ f_1(x) = 1 \text{ and } Z \subseteq \text{dom}(\Box^n \cdot (\text{the function arccos})) \text{ and } Z = \text{dom } f \text{ and } 1 < n \text{ and } f = \frac{n((\Box^{n-1}) \cdot (\text{the function arccos}))}{(\Box^2 \cdot (f_1 - f_2))}. \]

Then \[ \int_A f(x) dx = (-\Box^n \cdot (\text{the function arccos})) (\sup A) - (-\Box^n \cdot (\text{the function arccos})) (\inf A). \]

(21) Suppose \( A \subseteq Z \) and for every \( x \) such that \( x \in Z \) holds \( f_1(x) = 1 \) and \( Z \subseteq [-1, 1] \) and \( Z = \text{dom } f \) and \( f = (\text{the function arcsin}) + \frac{id_Z}{(\Box^2 \cdot (f_1 - f_2))}. \)

Then \[ \int_A f(x) dx = (id_Z (\text{the function arcsin})) (sup A) - (id_Z (\text{the function arcsin})) (inf A). \]

(22) Suppose \( A \subseteq Z \) and for every \( x \) such that \( x \in Z \) holds \( f_1(x) = 1 \) and \( Z \subseteq [-1, 1] \) and \( Z = \text{dom } f \) and \( f = (\text{the function arccos}) - \frac{id_Z}{(\Box^2 \cdot (f_1 - f_2))}. \)

Then \[ \int_A f(x) dx = (id_Z (\text{the function arccos})) (sup A) - (id_Z (\text{the function arccos})) (inf A). \]

(23) Suppose that

- (i) \( A \subseteq Z \),
- (ii) \( Z \subseteq [-1, 1] \),
- (iii) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = a \cdot x + b \) and \( f_2(x) = 1 \),
- (iv) \( Z = \text{dom } f \), and
- (v) \( f = a (\text{the function arcsin}) + \frac{f_1}{(\Box^2 \cdot (f_2 - f_2))}. \)

Then \[ \int_A f(x) dx = (f_1 (\text{the function arcsin})) (sup A) - (f_1 (\text{the function arcsin})) (inf A). \]

(24) Suppose that

- (i) \( A \subseteq Z \),
- (ii) \( Z \subseteq [-1, 1] \),
- (iii) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = a \cdot x + b \) and \( f_2(x) = 1 \),
- (iv) \( Z = \text{dom } f \), and
- (v) \( f = a (\text{the function arccos}) - \frac{f_1}{(\Box^2 \cdot (f_2 - f_2))}. \)

Then \[ \int_A f(x) dx = (f_1 (\text{the function arccos})) (sup A) - (f_1 (\text{the function arccos})) (inf A). \]

(25) Suppose that

- (i) \( A \subseteq Z \),
- (ii) for every \( x \) such that \( x \in Z \) holds \( g(x) = 1 \) and \( f_1(x) = \frac{x}{a} \) and \( f_1(x) > -1 \) and \( f_1(x) < 1 \),
- (iii) \( Z = \text{dom } f \),
- (iv) \( f \) is continuous on \( A \), and
(v) \( f = (\text{the function arcsin}) \cdot f_1 + \frac{id_Z}{a ((\square^n - f_1)(a - f_1)^2)} \).

Then \( \int_A f(x) dx = (id_Z ((\text{the function arcsin}) \cdot f_1))(sup A) - (id_Z ((\text{the function arcsec}) \cdot f_1))(inf A) \).

(26) Suppose that
(i) \( A \subseteq Z \),
(ii) for every \( x \) such that \( x \in Z \) holds \( g(x) = 1 \) and \( f_1(x) > -1 \) and \( f_1(x) < 1 \),
(iii) \( Z = \text{dom} f \),
(iv) \( f \) is continuous on \( A \), and
(v) \( f = (\text{the function arccos}) \cdot f_1 \).

Then \( \int_A f(x) dx = (id_Z ((\text{the function arccos}) \cdot f_1))(sup A) - (id_Z ((\text{the function arccos}) \cdot f_1))(inf A) \).

(27) Suppose \( A \subseteq Z \) and \( f = \frac{n ((\square^n - f_1))(\text{the function sin})}{(\square^n - f_1)(\text{the function cos})} \) and \( 1 \leq n \) and \( Z \subseteq \text{dom}((\square^n) \cdot (\text{the function tan})) \) and \( Z = \text{dom} f \). Then \( \int_A f(x) dx = (\text{sup A}) - ((\square^n) \cdot (\text{the function tan})) \).

(28) Suppose \( A \subseteq Z \) and \( f = \frac{n ((\square^n - f_1))(\text{the function cos})}{(\square^n - f_1)(\text{the function sin})} \) and \( 1 \leq n \) and \( Z \subseteq \text{dom}((\square^n) \cdot (\text{the function cot})) \) and \( Z = \text{dom} f \). Then \( \int_A f(x) dx = (\text{sup A}) - ((\square^n) \cdot (\text{the function cot})) \).

(29) Suppose that
(i) \( A \subseteq Z \),
(ii) \( Z \subseteq \text{dom}((\text{the function tan}) \cdot f_1) \),
(iii) \( f = ((\text{the function sin}) \cdot f_1)^2((\text{the function cos}) \cdot f_1)^2 \),
(iv) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = a \cdot x \) and \( a \neq 0 \), and
(v) \( Z = \text{dom} f \).

Then \( \int_A f(x) dx = (\frac{1}{a} ((\text{the function tan}) \cdot f_1) - id_Z)(sup A) - (\frac{1}{a} ((\text{the function tan}) \cdot f_1) - id_Z)(inf A) \).

(30) Suppose that
(i) \( A \subseteq Z \),
(ii) \( Z \subseteq \text{dom}((\text{the function cot}) \cdot f_1) \),
(iii) \( f = ((\text{the function cos}) \cdot f_1)^2((\text{the function sin}) \cdot f_1)^2 \),
(iv) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = a \cdot x \) and \( a \neq 0 \), and
(v) \( Z = \text{dom} f \).
Then \( \int_A f(x)\,dx = \left( \frac{1}{a} \right) ((\text{the function cot}) \cdot f_1) - \text{id}_Z)(\sup A) - \left( \frac{1}{a} \right) ((\text{the function cot}) \cdot f_1) - \text{id}_Z)(\inf A). \)

(31) Suppose that
(i) \( A \subseteq Z, \)
(ii) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = a \cdot x + b, \)
(iii) \( Z = \text{dom} f, \)
(iv) \( f = a \frac{\text{the function sin}}{\text{the function cos}} + \frac{f_1}{(\text{the function cos})^2}. \)

Then \( \int_A f(x)\,dx = (f_1 (\text{the function tan}))(\sup A) - (f_1 (\text{the function tan}))(\inf A). \)

(32) Suppose that
(i) \( A \subseteq Z, \)
(ii) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = a \cdot x + b, \)
(iii) \( Z = \text{dom} f, \)
(iv) \( f = a \frac{\text{the function cos}}{\text{the function sin}} - \frac{f_1}{(\text{the function sin})^2}. \)

Then \( \int_A f(x)\,dx = (f_1 (\text{the function cot}))(\sup A) - (f_1 (\text{the function cot}))(\inf A). \)

(33) Suppose that
(i) \( A \subseteq Z, \)
(ii) for every \( x \) such that \( x \in Z \) holds \( f(x) = \frac{(\text{the function sin})(x)^2}{(\text{the function cos})(x)^2}, \)
(iii) \( Z \subseteq \text{dom}((\text{the function tan})-\text{id}_Z), \)
(iv) \( Z = \text{dom} f, \)
(v) \( f \) is continuous on \( A. \)

Then \( \int_A f(x)\,dx = ((\text{the function tan})-\text{id}_Z)(\sup A) - ((\text{the function tan})-\text{id}_Z)(\inf A). \)

(34) Suppose that
(i) \( A \subseteq Z, \)
(ii) for every \( x \) such that \( x \in Z \) holds \( f(x) = \frac{(\text{the function cos})(x)^2}{(\text{the function sin})(x)^2}, \)
(iii) \( Z \subseteq \text{dom}(-\text{the function cot} - \text{id}_Z), \)
(iv) \( Z = \text{dom} f, \)
(v) \( f \) is continuous on \( A. \)

Then \( \int_A f(x)\,dx = (-\text{the function cot} - \text{id}_Z)(\sup A) - (-\text{the function cot} - \text{id}_Z)(\inf A). \)

(35) Suppose that
(i) \( A \subseteq Z, \)
(ii) for every \( x \) such that \( x \in Z \) holds \( f(x) = \frac{1}{x \cdot (1 + (\text{the function ln})(x)^2)} \) and \( (\text{the function ln})(x) > -1 \) and \( (\text{the function ln})(x) < 1, \)
(iii) \( Z \subseteq \text{dom}(\text{the function arctan}) \cdot (\text{the function ln}) \),
(iv) \( Z = \text{dom } f \), and
(v) \( f \) is continuous on \( A \).

Then \( \int_A f(x)dx = ((\text{the function arctan}) \cdot (\text{the function ln}))\,(\sup A) - ((\text{the function arctan}) \cdot (\text{the function ln}))\,(\inf A) \).

(36) Suppose that
(i) \( A \subseteq Z \),
(ii) for every \( x \) such that \( x \in Z \) holds \( f(x) = \frac{1}{x} \cdot (1+(\text{the function ln})(x)^2) \) and (the function ln)(x) > \(-1\) and (the function ln)(x) < \(1\),
(iii) \( Z \subseteq \text{dom}(\text{the function arccot}) \cdot (\text{the function ln}) \),
(iv) \( Z = \text{dom } f \), and
(v) \( f \) is continuous on \( A \).

Then \( \int_A f(x)dx = ((\text{the function arccot}) \cdot (\text{the function ln}))\,(\sup A) - ((\text{the function arccot}) \cdot (\text{the function ln}))\,(\inf A) \).

(37) Suppose that
(i) \( A \subseteq Z \),
(ii) for every \( x \) such that \( x \in Z \) holds \( f(x) = \frac{a}{\sqrt{1-(a \cdot x+b)^2}} \) and \( f_1(x) = a \cdot x + b \) and \( f_1(x) > -1 \) and \( f_1(x) < 1 \),
(iii) \( Z \subseteq \text{dom}(\text{the function arcsin}) \cdot f_1 \),
(iv) \( Z = \text{dom } f \), and
(v) \( f \) is continuous on \( A \).

Then \( \int_A f(x)dx = ((\text{the function arcsin}) \cdot f_1)\,(\sup A) - ((\text{the function arcsin}) \cdot f_1)\,(\inf A) \).

(38) Suppose that
(i) \( A \subseteq Z \),
(ii) for every \( x \) such that \( x \in Z \) holds \( f(x) = \frac{a}{\sqrt{1-(a \cdot x+b)^2}} \) and \( f_1(x) = a \cdot x + b \) and \( f_1(x) > -1 \) and \( f_1(x) < 1 \),
(iii) \( Z \subseteq \text{dom}(\text{the function arccos}) \cdot f_1 \),
(iv) \( Z = \text{dom } f \), and
(v) \( f \) is continuous on \( A \).

Then \( \int_A f(x)dx = (-((\text{the function arccos}) \cdot f_1))\,(\sup A) - (-((\text{the function arccos}) \cdot f_1))\,(\inf A) \).

(39) Suppose that \( A \subseteq Z \) and \( f_1 = g - f_2 \) and \( f_2 = \square^2 \) and for every \( x \) such that \( x \in Z \) holds \( f(x) = x \cdot (1-x^2)^{-\frac{1}{2}} \) and \( g(x) = 1 \) and \( f_1(x) > 0 \) and \( Z \subseteq \text{dom}(\square^3) \cdot f_1 \) and \( Z = \text{dom } f \) and \( f \) is continuous on \( A \). Then \( \int_A f(x)dx = \)
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(40) Suppose that $A \subseteq \mathbb{Z}$ and $g = f_1 - f_2$ and $f_2 = \Box^2$ and for every $x$ such that $x \in Z$ holds $f(x) = x \cdot (a^2 - x^2)^{-\frac{1}{2}}$ and $f_1(x) = a^2$ and $g(x) > 0$ and $Z \subseteq \text{dom}((\Box^2) \cdot g)$ and $Z = \text{dom} f$ and $f$ is continuous on $A$. Then

$$\int_A f(x)dx = \left(-\left(\frac{1}{a^2} \cdot f_1\right)\right)\left(\sup A\right) - \left(-\left(\frac{1}{a^2} \cdot f_1\right)\right)\left(\inf A\right).$$

(41) Suppose that

(i) $A \subseteq \mathbb{Z}$,

(ii) $n > 0$,

(iii) for every $x$ such that $x \in \mathbb{Z}$ holds $f(x) = \frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}}$ and $(\text{the function sin})(x) \neq 0$,

(iv) $Z \subseteq \text{dom}(\Box^n) \cdot \frac{1}{\text{the function sin}}$,

(v) $Z = \text{dom} f$, and

(vi) $f$ is continuous on $A$.

Then

$$\int_A f(x)dx = \left(\left(-\frac{1}{n}\right)\left(\frac{1}{\text{the function sin}}\right)\right)\left(\sup A\right) - \left(\left(-\frac{1}{n}\right)\left(\frac{1}{\text{the function sin}}\right)\right)\left(\inf A\right).$$

(42) Suppose that

(i) $A \subseteq \mathbb{Z}$,

(ii) $n > 0$,

(iii) for every $x$ such that $x \in \mathbb{Z}$ holds $f(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}}$ and $(\text{the function cos})(x) \neq 0$,

(iv) $Z \subseteq \text{dom}(\Box^n) \cdot \frac{1}{\text{the function cos}}$,

(v) $Z = \text{dom} f$, and

(vi) $f$ is continuous on $A$.

Then

$$\int_A f(x)dx = \left(\frac{1}{n}\left(\frac{1}{\text{the function cos}}\right)\right)\left(\sup A\right) - \left(\frac{1}{n}\left(\frac{1}{\text{the function cos}}\right)\right)\left(\inf A\right).$$

(43) Suppose that $A \subseteq \mathbb{Z}$ and $f = \frac{1}{g_1(x)}$ and $g_2 = \Box^2$ and for every $x$ such that $x \in Z$ holds $f(x) = (1+x^2)^{-1} \cdot (\text{the function arccot})(x)$ and $g_1(x) = 1$ and $g_2(x) > 0$ and $Z = \text{dom} f$. Then

$$\int_A f(x)dx = \left(-\left(\text{the function ln}\right) \cdot (\text{the function arccot})\right)\left(\sup A\right) - \left(-\left(\text{the function ln}\right) \cdot (\text{the function arccot})\right)\left(\inf A\right).$$

(44) Suppose that

(i) $A \subseteq \mathbb{Z}$,

(ii) $Z \subseteq [\!-1, 1[,$

(iii) for every $x$ such that $x \in Z$ holds $\text{the function arcsin}(x) > 0$ and $f_1(x) = 1$,
(iv) \( Z \subseteq \text{dom}((\text{the function } \ln) \cdot (\text{the function } \arcsin)) \),
(v) \( Z = \text{dom } f \), and
(vi) \( f = \frac{1}{((□\overline{2}) \cdot (f_1 - □\overline{2}) \cdot (\text{the function } \arcsin))} \).

Then \( \int_A f(x)dx = ((\text{the function } \ln) \cdot (\text{the function } \arcsin))(\sup A) - ((\text{the function } \ln) \cdot (\text{the function } \arcsin))(\inf A) \).

Suppose that
(i) \( A \subseteq Z \),
(ii) \( Z \subseteq [-1, 1] \),
(iii) for every \( x \) such that \( x \in Z \) holds \( f_1(x) = 1 \) and \( (\text{the function } \arccos)(x) > 0 \),
(iv) \( Z \subseteq \text{dom}((\text{the function } \ln) \cdot (\text{the function } \arccos)) \),
(v) \( Z = \text{dom } f \), and
(vi) \( f = \frac{1}{((□\overline{2}) \cdot (f_1 - □\overline{2}) \cdot (\text{the function } \arccos))} \).

Then \( \int_A f(x)dx = ((- (\text{the function } \ln) \cdot (\text{the function } \arccos))(\sup A) - ((- (\text{the function } \ln) \cdot (\text{the function } \arccos))(\inf A) \).

References


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