Basic Properties of Periodic Functions

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Summary. In this article we present definitions, basic properties and some examples of periodic functions according to [5].

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The papers [2], [6], [3], [10], [11], [9], [8], [1], [4], and [7] provide the terminology and notation for this paper.

1. BASIC PROPERTIES OF A PERIOD OF A FUNCTION

We use the following convention: $x, t, t_1, t_2, r, a, b$ are real numbers and $F, G$ are partial functions from $\mathbb{R}$ to $\mathbb{R}$.

Let $F$ be a partial function from $\mathbb{R}$ to $\mathbb{R}$ and let $t$ be a real number. We say that $t$ is a period of $F$ if and only if:

(Def. 1) $t \neq 0$ and for every $x$ holds $x \in \text{dom } F$ iff $x + t \in \text{dom } F$ and if $x \in \text{dom } F$, then $F(x) = F(x + t)$.

Let $F$ be a partial function from $\mathbb{R}$ to $\mathbb{R}$. We say that $F$ is periodic if and only if:

(Def. 2) There exists $t$ which is a period of $F$.

We now state a number of propositions:

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One can check the following observations:

(1) \( t \) is a period of \( F \) iff \( t \neq 0 \) and for every \( x \) such that \( x \in \text{dom} \ F \) holds \( x + t, x - t \in \text{dom} \ F \) and \( F(x) = F(x + t) \).

(2) If \( t \) is a period of \( F \) and a period of \( G \), then \( t \) is a period of \( F + G \).

(3) If \( t \) is a period of \( F \) and a period of \( G \), then \( t \) is a period of \( F - G \).

(4) If \( t \) is a period of \( F \) and a period of \( G \), then \( t \) is a period of \( FG \).

(5) If \( t \) is a period of \( F \) and a period of \( G \), then \( t \) is a period of \( F/G \).

(6) If \( t \) is a period of \( F \), then \( t \) is a period of \( -F \).

(7) If \( t \) is a period of \( F \), then \( t \) is a period of \( rF \).

(8) If \( t \) is a period of \( F \), then \( t \) is a period of \( r + F \).

(9) If \( t \) is a period of \( F \), then \( t \) is a period of \( F - r \).

(10) If \( t \) is a period of \( F \), then \( t \) is a period of \( |F| \).

(11) If \( t \) is a period of \( F \), then \( t \) is a period of \( F^{-1} \).

(12) If \( t \) is a period of \( F \), then \( t \) is a period of \( F^2 \).

(13) If \( t \) is a period of \( F \), then for every \( x \) such that \( x \in \text{dom} \ F \) holds \( F(x) = F(x - t) \).

(14) If \( t \) is a period of \( F \), then \( -t \) is a period of \( F \).

(15) If \( t_1 \) is a period of \( F \) and \( t_2 \) is a period of \( F \) and \( t_1 + t_2 \neq 0 \), then \( t_1 + t_2 \) is a period of \( F \).

(16) If \( t_1 \) is a period of \( F \) and \( t_2 \) is a period of \( F \) and \( t_1 - t_2 \neq 0 \), then \( t_1 - t_2 \) is a period of \( F \).

(17) Suppose \( t \neq 0 \) and for every \( x \) such that \( x \in \text{dom} \ F \) holds \( x + t, x - t \in \text{dom} \ F \) and \( F(x + t) = F(x - t) \). Then \( 2 \cdot t \) is a period of \( F \) and \( F^2 \) is periodic.

(18) Suppose \( t_1 + t_2 \neq 0 \) and for every \( x \) such that \( x \in \text{dom} \ F \) holds \( x + t_1, x - t_1, x + t_2, x - t_2 \in \text{dom} \ F \) and \( F(x + t_1) = F(x - t_2) \). Then \( t_1 + t_2 \) is a period of \( F \) and \( F^2 \) is periodic.

(19) Suppose \( t_1 - t_2 \neq 0 \) and for every \( x \) such that \( x \in \text{dom} \ F \) holds \( x + t_1, x - t_1, x + t_2, x - t_2 \in \text{dom} \ F \) and \( F(x + t_1) = F(x - t_2) \). Then \( t_1 - t_2 \) is a period of \( F \) and \( F^2 \) is periodic.

(20) Suppose \( t \neq 0 \) and for every \( x \) such that \( x \in \text{dom} \ F \) holds \( x + t, x - t \in \text{dom} \ F \) and \( F(x + t) = F(x)^{-1} \). Then \( 2 \cdot t \) is a period of \( F \) and \( F^{-1} \) is periodic.

Let us observe that there exists a partial function from \( \mathbb{R} \) to \( \mathbb{R} \) which is periodic.

Let \( F \) be a periodic partial function from \( \mathbb{R} \) to \( \mathbb{R} \). One can check that \( -F \) is periodic.

Let \( F \) be a periodic partial function from \( \mathbb{R} \) to \( \mathbb{R} \) and let \( r \) be a real number. One can check the following observations:

\* \( r \cdot F \) is periodic,

\* \( r + F \) is periodic, and
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F − r is periodic.

Let F be a periodic partial function from \( \mathbb{R} \) to \( \mathbb{R} \). One can check the following observations:

* \( |F| \) is periodic,
* \( F^{-1} \) is periodic, and
* \( F^2 \) is periodic.

2. Some Examples

Let us note that the function sin is periodic and the function cos is periodic.

We now state two propositions:

(21) For every element \( k \) of \( \mathbb{N} \) holds \( 2 \cdot \pi \cdot (k + 1) \) is a period of the function sin.

(22) For every element \( k \) of \( \mathbb{N} \) holds \( 2 \cdot \pi \cdot (k + 1) \) is a period of the function cos.

Let us observe that the function cosec is periodic and the function sec is periodic.

We now state two propositions:

(23) For every element \( k \) of \( \mathbb{N} \) holds \( 2 \cdot \pi \cdot (k + 1) \) is a period of the function sec.

(24) For every element \( k \) of \( \mathbb{N} \) holds \( 2 \cdot \pi \cdot (k + 1) \) is a period of the function cosec.

Let us mention that the function tan is periodic and the function cot is periodic.

Next we state a number of propositions:

(25) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of the function tan.

(26) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of the function cot.

(27) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of \( |\text{the function sin}| \).

(28) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of \( |\text{the function cos}| \).

(29) For every element \( k \) of \( \mathbb{N} \) holds \( \frac{\pi}{2} \cdot (k + 1) \) is a period of \( |\text{the function sin}| + |\text{the function cos}| \).

(30) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of \( (\text{the function sin})^2 \).

(31) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of \( (\text{the function cos})^2 \).

(32) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of \( (\text{the function sin}) \) \( (\text{the function cos}) \).

(33) For every element \( k \) of \( \mathbb{N} \) holds \( \pi \cdot (k + 1) \) is a period of \( (\text{the function cos}) \) \( (\text{the function sin}) \).

(34) For every element \( k \) of \( \mathbb{N} \) holds \( 2 \cdot \pi \cdot (k + 1) \) is a period of \( b + a \) (the function sin).

(35) For every element \( k \) of \( \mathbb{N} \) holds \( 2 \cdot \pi \cdot (k + 1) \) is a period of \( a \) (the function sin) \( -b \).
(36) For every element $k$ of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k + 1)$ is a period of $b + a$ (the function $\cos$).

(37) For every element $k$ of $\mathbb{N}$ holds $2 \cdot \pi \cdot (k + 1)$ is a period of $a$ (the function $\cos$) $- b$.

(38) If $\text{dom } F = \mathbb{R}$ and for every real number $x$ holds $F(x) = a$, then for every element $k$ of $\mathbb{N}$ holds $k + 1$ is a period of $F$.

References


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