Cell Petri Net Concepts

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Summary. Based on the Petri net definitions and theorems already formalized in [8], with this article, we developed the concept of “Cell Petri Nets”. It is based on [9]. In a cell Petri net we introduce the notions of colors and colored states of a Petri net, connecting mappings for linking two Petri nets, firing rules for transitions, and the synthesis of two or more Petri nets.

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The papers [11], [12], [6], [13], [14], [10], [8], [2], [5], [3], [4], [7], and [1] provide the terminology and notation for this paper.

1. Preliminaries: Thin Cylinder, Locus

Let $A$ be a non empty set, let $B$ be a set, let $B_1$ be a set, and let $y_1$ be a function from $B_1$ into $A$. Let us assume that $B_1 \subseteq B$. The functor $\text{cylinder}_0(A, B, B_1, y_1)$ yields a non empty subset of $A^B$ and is defined by:

$$\text{cylinder}_0(A, B, B_1, y_1) = \{ y : B \to A : y|B_1 = y_1 \}.$$  

Let $A$ be a non empty set and let $B$ be a set. A non empty subset of $A^B$ is said to be a thin cylinder of $A$ and $B$ if:

$$\text{cylinder}_0(A, B, B_1, y_1) = \{ y : B \to A : y|B_1 = y_1 \}.$$  

There exists a subset $B_1$ of $B$ and there exists a function $y_1$ from $B_1$ into $A$ such that $B_1$ is finite and it = $\text{cylinder}_0(A, B, B_1, y_1)$.

The following propositions are true:
Let $A$ be a non empty set, $B$ be a set, and $D$ be a thin cylinder of $A$ and $B$. Then there exists a subset $B_1$ of $B$ and there exists a function $y_1$ from $B_1$ into $A$ such that $B_1$ is finite and $D = \{ y : B \to A : y[B_1 = y_1] \}$.

Let $A_1$, $A_2$ be non empty sets, $B$ be a set, and $D_1$ be a thin cylinder of $A_1$ and $B$. If $A_1 \subseteq A_2$, then there exists a thin cylinder $D_2$ of $A_2$ and $B$ such that $D_1 \subseteq D_2$.

Let $A$ be a non empty set and let $B$ be a set. The thin cylinders of $A$ and $B$ constitute a non empty family of subsets of $A^B$ defined by:

\[(\text{Def. 3}) \quad \text{The thin cylinders of } A \text{ and } B = \{ D \subseteq A^B : D \text{ is a thin cylinder of } A \text{ and } B \}.\]

We now state three propositions:

\[(\text{Def. 4}) \quad \text{Let } A \text{ be a non trivial set, } B \text{ be a set, } B_2 \text{ be a set, } y_2 \text{ be a function from } B_2 \text{ into } A, \ B_3 \text{ be a set, and } y_3 \text{ be a function from } B_3 \text{ into } A. \text{ If } B_2 \subseteq B \text{ and } B_3 \subseteq B \text{ and } \text{cylinder}_0(A, B, B_2, y_2) = \text{cylinder}_0(A, B, B_3, y_3), \text{ then } B_2 = B_3 \text{ and } y_2 = y_3.\]

\[(\text{Def. 3}) \quad \text{Let } A_1, A_2 \text{ be non empty sets and } B_4, B_5 \text{ be sets. Suppose } A_1 \subseteq A_2 \text{ and } B_4 \subseteq B_5. \text{ Then there exists a function } F \text{ from the thin cylinders of } A_1 \text{ and } B_4 \text{ into the thin cylinders of } A_2 \text{ and } B_5 \text{ such that for every set } x \text{ if } x \in \text{ the thin cylinders of } A_1 \text{ and } B_4, \text{ then there exists a subset } B_1 \text{ of } B_4 \text{ and there exists a function } y_2 \text{ from } B_1 \text{ into } A_1 \text{ and there exists a function } y_3 \text{ from } B_1 \text{ into } A_2 \text{ such that } B_1 \text{ is finite and } y_2 = y_3 \text{ and } x = \text{cylinder}_0(A_1, B_4, B_1, y_2) \text{ and } F(x) = \text{cylinder}_0(A_2, B_5, B_1, y_3).\]

\[(\text{Def. 4}) \quad \text{Let } A_1, A_2 \text{ be non empty sets and } B_4, B_5 \text{ be sets. Then there exists a function } G \text{ from the thin cylinders of } A_2 \text{ and } B_5 \text{ into the thin cylinders of } A_1 \text{ and } B_4 \text{ such that for every set } x \text{ if } x \in \text{ the thin cylinders of } A_2 \text{ and } B_5, \text{ then there exists a subset } B_3 \text{ of } B_5 \text{ and there exists a subset } B_2 \text{ of } B_4 \text{ and there exists a function } y_2 \text{ from } B_2 \text{ into } A_1 \text{ and there exists a function } y_3 \text{ from } B_3 \text{ into } A_2 \text{ such that } B_2 \text{ is finite and } B_3 \text{ is finite and } B_2 = B_4 \cap B_3 \cap y_3^{-1}(A_1) \text{ and } y_2 = y_3|B_2 \text{ and } x = \text{cylinder}_0(A_2, B_5, B_3, y_3) \text{ and } G(x) = \text{cylinder}_0(A_1, B_4, B_2, y_2).\]

Let $A_1$, $A_2$ be non trivial sets and let $B_4$, $B_5$ be sets. Let us assume that there exist sets $x$, $y$ such that $x \neq y$ and $x, y \in A_1$ and $A_1 \subseteq A_2$ and $B_4 \subseteq B_5$. The functor Extcylinders$(A_1, B_4, A_2, B_5)$ yielding a function from the thin cylinders of $A_1$ and $B_4$ into the thin cylinders of $A_2$ and $B_5$ is defined by the condition (Def. 4).

\[(\text{Def. 4}) \quad \text{Let } x \text{ be a set. Suppose } x \in \text{ the thin cylinders of } A_1 \text{ and } B_4. \text{ Then there exists a subset } B_1 \text{ of } B_1 \text{ and there exists a function } y_2 \text{ from } B_1 \text{ into } A_1 \text{ and there exists a function } y_3 \text{ from } B_1 \text{ into } A_2 \text{ such that } B_1 \text{ is finite and } y_2 = y_3 \text{ and } x = \text{cylinder}_0(A_1, B_4, B_1, y_2) \text{ and } (\text{Extcylinders}(A_1, B_4, A_2, B_5))(x) = \text{cylinder}_0(A_2, B_5, B_1, y_3).\]
Let $A_1$ be a non empty set, let $A_2$ be a non trivial set, and let $B_4$, $B_5$ be sets. Let us assume that $A_1 \subseteq A_2$ and $B_4 \subseteq B_5$. The functor $\text{Ristcylinders}(A_1, B_4, A_2, B_5)$ yields a function from the thin cylinders of $A_2$ and $B_5$ into the thin cylinders of $A_1$ and $B_4$ and is defined by the condition (Def. 5).

(Def. 5) Let $x$ be a set. Suppose $x \in$ the thin cylinders of $A_2$ and $B_5$. Then there exists a subset $B_3$ of $B_5$ and there exists a subset $B_2$ of $B_4$ and there exists a function $y_2$ from $B_2$ into $A_1$ and there exists a function $y_3$ from $B_3$ into $A_2$ such that $B_2$ is finite and $B_3$ is finite and $B_2 = B_4 \cap B_3 \cap y_3^{-1}(A_1)$ and $y_2 = y_3\mid B_2$ and $x = \text{cylinder}_0(A_2, B_5, B_3, y_3)$ and $(\text{Ristcylinders}(A_1, B_4, A_2, B_5))(x) = \text{cylinder}_0(A_1, B_4, B_2, y_2)$.

Let $A$ be a non trivial set, let $B$ be a set, and let $D$ be a thin cylinder of $A$ and $B$. The functor $\text{loc} D$ yielding a finite subset of $B$ is defined by the condition (Def. 6).

(Def. 6) There exists a subset $B_1$ of $B$ and there exists a function $y_1$ from $B_1$ into $A$ such that $B_1$ is finite and $D = \{y : B \to A : y\mid B_1 = y_1\}$ and $\text{loc} D = B_1$.

2. Colored Petri Nets

Let $A_1$, $A_2$ be non trivial sets, let $B_4$, $B_5$ be sets, let $C_1$, $C_2$ be non trivial sets, let $D_1$, $D_2$ be sets, and let $F$ be a function from the thin cylinders of $A_1$ and $B_4$ into the thin cylinders of $C_1$ and $D_1$. The functor $\text{CylinderFunc}(A_1, B_4, A_2, B_5, C_1, D_1, C_2, D_2, F)$ yielding a function from the thin cylinders of $A_2$ and $B_5$ into the thin cylinders of $C_2$ and $D_2$ is defined as follows:

(Def. 7) $\text{CylinderFunc}(A_1, B_4, A_2, B_5, C_1, D_1, C_2, D_2, F) =$

$\text{Extcylinders}(C_1, D_1, C_2, D_2) \cdot F \cdot \text{Ristcylinders}(A_1, B_4, A_2, B_5)$.

We consider colored place/transition net structures as extensions of place/transition net structure as systems

$\langle \text{places}, \text{transitions}, \text{S-T arcs}, \text{T-S arcs}, \text{a colored set, a firing-rule } \rangle,$

where the places and the transitions constitute non empty sets, the S-T arcs constitute a non empty relation between the places and the transitions, the T-S arcs constitute a non empty relation between the transitions and the places, the colored set is a non empty finite set, and the firing-rule is a function.

Let $C_3$ be a colored place/transition net structure and let $t_0$ be a transition of $C_3$. We say that $t_0$ is outbound if and only if:

(Def. 8) $\{t_0\} = \emptyset$.

Let $C_4$ be a colored place/transition net structure. The functor $\text{Outbds} C_4$ yielding a subset of the transitions of $C_4$ is defined by:

(Def. 9) $\text{Outbds} C_4 = \{x : x \text{ ranges over transitions of } C_4 : x \text{ is outbound}\}$. 
Let $C_3$ be a colored place/transition net structure. We say that $C_3$ is colored-PT-net-like if and only if the conditions (Def. 10) are satisfied.

(Def. 10)

(i) dom (the firing-rule of $C_3$) $\subseteq$ (the transitions of $C_3$) \ Outbds $C_3$, and

(ii) for every transition $t$ of $C_3$ such that $t \in$ dom (the firing-rule of $C_3$)
there exists a non empty subset $C_5$ of the colored set of $C_3$ and there exists a subset $I$ of $^*\{t\}$ and there exists a subset $O$ of $\{t\}$ such that (the firing-rule of $C_3$)($t$) is a function from the thin cylinders of $C_5$ and $I$ into the thin cylinders of $C_5$ and $O$.

We now state two propositions:

(6) Let $C_3$ be a colored place/transition net structure and $t$ be a transition of $C_3$. Suppose $C_3$ is colored-PT-net-like and $t \in$ dom (the firing-rule of $C_3$). Then there exists a non empty subset $C_5$ of the colored set of $C_3$ and there exists a subset $I$ of $^*\{t\}$ and there exists a subset $O$ of $\{t\}$ such that (the firing-rule of $C_3$)($t$) is a function from the thin cylinders of $C_5$ and $I$ into the thin cylinders of $C_5$ and $O$.

(7) Let $C_4$, $C_6$ be colored place/transition net structures, $t_1$ be a transition of $C_4$, and $t_2$ be a transition of $C_6$. Suppose that

(i) the places of $C_4$ $\subseteq$ the places of $C_6$,

(ii) the transitions of $C_4$ $\subseteq$ the transitions of $C_6$,

(iii) the S-T arcs of $C_4$ $\subseteq$ the S-T arcs of $C_6$,

(iv) the T-S arcs of $C_4$ $\subseteq$ the T-S arcs of $C_6$, and

(v) $t_1 = t_2$.

Then $^*\{t_1\}$ $\subseteq$ $^*\{t_2\}$ and $\{t_1\}$ $\subseteq$ $\{t_2\}$.

One can verify that there exists a colored place/transition net structure which is strict and colored-PT-net-like.

A colored place/transition net is a colored-PT-net-like colored place/transition net structure.

3. Color Counts of CPNT

Let $C_4$, $C_6$ be colored place/transition net structures. We say that $C_4$ misses $C_6$ if and only if:

(Def. 11) (The places of $C_4$) $\cap$ (the places of $C_6$) = $\emptyset$ and (the transitions of $C_4$) $\cap$ (the transitions of $C_6$) = $\emptyset$.

Let us note that the predicate $C_4$ misses $C_6$ is symmetric.

4. Colored States of CPNT

Let $C_4$ be a colored place/transition net structure and let $C_6$ be a colored place/transition net structure. Connecting mapping of $C_4$ and $C_6$ is defined by the condition (Def. 12).
(Def. 12) There exists a function $O_{12}$ from $\text{Outbds} C_4$ into the places of $C_6$ and there exists a function $O_{21}$ from $\text{Outbds} C_6$ into the places of $C_4$ such that $\text{it} = \{O_{12}, O_{21}\}$.

5. Outbound Transitions of CPNT

Let $C_4, C_6$ be colored place/transition nets and let $O$ be a connecting mapping of $C_4$ and $C_6$. Connecting firing rule of $C_4$, $C_6$, and $O$ is defined by the condition (Def. 13).

(Def. 13) There exist functions $q_{12}, q_{21}$ and there exists a function $O_{12}$ from $\text{Outbds} C_4$ into the places of $C_6$ and there exists a function $O_{21}$ from $\text{Outbds} C_6$ into the places of $C_4$ such that

(i) $O = \{O_{12}, O_{21}\}$,
(ii) $\text{dom} q_{12} = \text{Outbds} C_4$,
(iii) $\text{dom} q_{21} = \text{Outbds} C_6$,
(iv) for every transition $t_3$ of $C_4$ such that $t_3$ is outbound holds $q_{12}(t_3)$ is a function from the thin cylinders of the colored set of $C_4$ and $\ast \{t_3\}$ into the thin cylinders of the colored set of $C_4$ and $O_{12} \circ t_3$,
(v) for every transition $t_4$ of $C_6$ such that $t_4$ is outbound holds $q_{21}(t_4)$ is a function from the thin cylinders of the colored set of $C_6$ and $\ast \{t_4\}$ into the thin cylinders of the colored set of $C_6$ and $O_{21} \circ t_4$, and
(vi) $\text{it} = \{q_{12}, q_{21}\}$.

6. Connecting Mapping for CPNT1, CPNT2

Let $C_4, C_6$ be colored place/transition nets, let $O$ be a connecting mapping of $C_4$ and $C_6$, and let $q$ be a connecting firing rule of $C_4$, $C_6$, and $O$. Let us assume that $C_4$ misses $C_6$. The functor $\text{synthesis}(C_4, C_6, O, q)$ yielding a strict colored place/transition net is defined by the condition (Def. 14).

(Def. 14) There exist functions $q_{12}, q_{21}$ and there exists a function $O_{12}$ from $\text{Outbds} C_4$ into the places of $C_6$ and there exists a function $O_{21}$ from $\text{Outbds} C_6$ into the places of $C_4$ such that

$O = \{O_{12}, O_{21}\}$ and $\text{dom} q_{12} = \text{Outbds} C_4$ and $\text{dom} q_{21} = \text{Outbds} C_6$ and for every transition $t_3$ of $C_4$ such that $t_3$ is outbound holds $q_{12}(t_3)$ is a function from the thin cylinders of the colored set of $C_4$ and $\ast \{t_3\}$ into the thin cylinders of the colored set of $C_4$ and $O_{12} \circ t_3$ and for every transition $t_4$ of $C_6$ such that $t_4$ is outbound holds $q_{21}(t_4)$ is a function from the thin cylinders of the colored set of $C_6$ and $\ast \{t_4\}$ into the thin cylinders of the colored set of $C_6$ and $O_{21} \circ t_4$ and $\text{it} = \{q_{12}, q_{21}\}$ and the places of synthesis$(C_4, C_6, O, q) = (\text{the places of } C_4) \cup (\text{the places of } C_6)$ and the
transitions of synthesis($C_4, C_6, O, q$) = (the transitions of $C_4$)$\cup$(the transitions of $C_6$) and the S-T arcs of synthesis($C_4, C_6, O, q$) = (the S-T arcs of $C_4$)$\cup$(the S-T arcs of $C_6$) and the T-S arcs of synthesis($C_4, C_6, O, q$) = (the T-S arcs of $C_4$)$\cup$(the T-S arcs of $C_6$)$\cup O_{12}\cup O_{21}$ and the colored set of synthesis($C_4, C_6, O, q$) = (the colored set of $C_4$)$\cup$(the colored set of $C_6$) and the firing-rule of synthesis($C_4, C_6, O, q$) = (the firing-rule of $C_4$)$\vdash$(the firing-rule of $C_6$)$\vdash q_{12}+q_{21}$.

REFERENCES


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