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COMPARISON OF THE TAILS OF MARKET RETURN DISTRIBUTIONS

Summary

The aim of this study is to analyze the tails of the distributions of stock market returns and to compare the differences between them. It is a well-established fact that the vast majority of stock market return distributions exhibit fat tails (a bigger probability of extreme outcomes than in the case of the normal probability). Apart from that, there seems to be a popular opinion that most market returns are negatively skewed with a fatter left tail. The study utilizes two methods for comparing the tails of a distribution. A simple approach based on the sample kurtosis, with a modification that allows for the calculation of kurtosis separately for the right and the left tail of a single distribution and a more complex approach based on the maximum likelihood fitting of the Generalized Pareto Distribution to both tails of standardized return distributions. The second approach is based on the assumptions of the Extreme Value Theory (EVT) and the Pickands-Balkema-de Haan theorem. Both approaches provide similar conclusions. Results suggest that whether the left or the right tail of the return distribution is bigger varies from market to market. All four major equity indices of the Polish Warsaw Stock Exchange exhibited a fatter left tail. However, in the whole sample it was actually more common for the right tail to be heavier, with 12 indices out of 20 exhibiting a fatter right tail than the left. The sample kurtosis indicated that all stock market return's distributions were heavy tailed, whereas the estimates of Generalized Pareto Distribution parameters did indicate standard or thin tails in two cases. Statistical tests indicate that the differences between the tails of stock market distributions are not statistically significant

Key words: Stock-market returns, fat tails, Extreme Value Theory, Generalized Pareto Distribution

1. Introduction

Financial markets constitute a very important element of the developed economies and can be seen as an important source of information. Information provided by the financial markets is used to evaluate the condition of enterprises and often the condition of the economy as a whole. One moment when the information from the financial markets can have the biggest actual impact on the economy is during rapid unexpected market movements, as in the case of the outbreaks of financial crises. The probability of extreme (and therefore rare) events is captured by the tails of probability distributions. Therefore, the analysis of the tails of stock market return probability distributions is an important issue in the study of the microstructure of financial markets. When a classic investment of purchasing a stock is considered the tails of

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stock market returns represent the probabilities of worst and best-case scenarios. The left tail represents the probability of an extreme loss and the right tail represents the probability of an abnormally high gain. This is not always so with more complicated investment strategies which employ short selling and/or derivative instruments. Despite that the tails of financial return distributions are a key element of the investment and market-risk analysis.

Among the scientists and investors alike there exists a set of the so-called stylized facts on the shape of the probability distributions of market returns. For example: daily expected return is approximately equal to zero, the distribution has a negative skew with the highest probability of a daily return slightly above the expected value (mode above zero), both tails of the distribution are heavier than those of the normal distribution [Cont, 2001, p. 224], the left tail of the of the distribution is heavier than the right one [Jondeau, Rockinger, 2003, p. 560; Bali et al., 2013, p. 1]. Some of the above facts are more firmly established than the others. Currently, there seems to be little or no doubt that the probability distribution of stock market returns is leptokurtic (has fatter tails) with respect to the normal distribution. However, when it comes to the skewness and the difference between the left and the right tail, the body of available empirical research cannot be considered conclusive [Jondeau, Rockinger, 2003 p. 560], [Wen, Yang 2009, p. 360]. The observation that busts are more common than the booms is sometimes given as the explanation for the concept that the left tail is fatter than the right. This would mean that in the case of a classic investment it is more probable to incur an extreme loss than an exceptionally abnormal gain. However as short selling and derivative instruments are becoming more common the left tail no longer represents only the probability of a loss.

The aim of this paper is to analyze the tails of the distributions of stock market returns and to compare the differences between them and especially to establish whether it is more common for the left tail to be heavier than the right one. This study tests the hypothesis that one tail of the stock return probability is heavier than the other based on a sample of 20 time series of daily logarithmic stock market returns from the period ranging from 1 January 2004 until 1 April 2014. The following indices were included in this study: WIG, WIG20, mWIG40 and MiS80 indices of the Polish Warsaw Stock Exchange; and S&P 500 (U.S.), Nikkei 225 (Japan), All Ordinaries Index (Australia), Bovespa Index (Brazil), SAX Index (Slovakia), PX Index (Czech Republic), OSE All Share Index (Norway), OMX Vilnius Index (Lithuania), OMX Tallinn Index (Estonia), OMX Riga Index (Latvia), OMX Helsinki Index (Finland), IBEX Index (Spain), FTSE 250 (United Kingdom), DAX Index (Germany), CAC40 (France), and BUX Index (Hungary). The study utilizes two methods for comparing the tails of the distribution. A simple approach based on the sample kurtosis, with a small modification that allows for the calculation of kurtosis separately for both tails of the distribution and a more sophisticated approach based on the maximum likelihood fitting of the Generalized Pareto Distribution to both tails of standardized return distributions. The second approach is based on the assumptions of the Extreme Value Theory (EVT) and the Pickands-Balkema-de Haan theorem [Balkema, de Haan, 1974; Pickands, 1975]. The obtained results suggest that there is no statistically significant difference between the left and the right tail of the distribution.

2. Tails of stock market return distributions

First works that demonstrated that tails of stock market returns are fatter than those of a normal distribution were published by Mandelbrot [Mandelbrot, 1963] and Fama [Fama, 1965]. Fatter tails of stock market returns mean that extreme market movements are more probable than other extreme events that have a Gaussian probability distribution. Most investors hold the opinion that the left tail of the distribution is heavier than the right one mostly due to the observation of higher autocorrelation of negative returns during bursting stock-bubbles [Jondeau, Rockinger, 2003 p. 2]. Extreme drops in market asset prices tend to follow one another, whereas during the bull market, the prices rise gradually and extreme jumps happen only occasionally and are usually separated events. There are many more possible explanations on why the left tail should be heavier than the right one. Campbell and Hentschel [Campbell, Hentschel, 1992] argue that news, whether good or bad tend to come in clusters and hence cause a rise in market volatility which in turn rises the risk premium and has a negative effect on stock prices. This effect has a diminishing influence on positive returns resulting from good news and a stimulating effect on negative returns caused by bad news. A different explanation relies on the arguments of the Prospect Theory of Kahneman and Tversky [Kahneman, Tversky, 1979]. The asymmetry of tails may be a result of differences in treatment of gains and losses by the investors, due to asymmetry in preferences and utility [Bali et al., 2013 p. 1]. In a study by Thurner, Farmer and Geanakoplos, the authors demonstrate how leveraged investing by large hedge funds can be responsible for fat tails in general [Thurner et al., 2010]. A recent argument for why left tails can be fatter than the right ones comes as a result of experiences from the recent financial crisis. Many papers study the tail dependencies among different markets, a phenomenon of extreme movements in prices on one market causing extreme movements in other markets. Studies by Jondeau [Jondeau, 2010] and Bollerslev, Todorov and Li [Bollerslev et al., 2011] among others show empirical results for asymmetry of tail dependencies in some markets with extreme negative price changes being more interdependent across different markets. In an extensive study Jondeau and Rockinger [Jondeau, Rockinger, 2003 p. 577], show that, although the left tail tends to be heavier in empirical data, for many markets the difference between tails of stock market returns is not statistically significant. Some of the above explanations for why the left tail of stock markets returns is heavier than the right one rely on the fact that the left tail is associated with the fact that the left tail generally represents losses and the right tail generally represents gains.

3. Data and research methodology

Data for this study was downloaded from the <http://stooq.pl> service and consists of daily logarithmic returns for 20 stock market indices from the period ranging from 1 January 2004 until 1 April 2014 what gives the average of 2580 observations per index. Returns were calculated from the daily closing prices. Four of the indices are from the Polish Warsaw Stock Exchange, further twelve indices come from other European

markets and the remaining four indices are from the markets outside of Europe. The studied indices are listed in table 1.

TABLE 1.

Stock-market indices included in the study

Polish	Outside Europe	European		
WIG	S&P 500 - U.S.A	SAX Index - Slovakia	OMX Tallinn Index - Estonia	FTSE 250 – United Kingdom
WIG 20	Nikkei 225 - Japan	PX Index – Czech Republic	OMX Riga Index - Latvia	DAX Index - Germany
mWIG40	All Ordinaries Index - Australia	OSE All Share Index - Norway	OMX Helsinki Index - Finland	CAC 40 - France
MiS80	Bovespa Index - Brazil	OMX Vilnius Index - Lithuania	IBEX Index - Spain	BUX Index - Hungary

Source: own elaboration

The most popular measure of the thickness of tails of a distribution is the kurtosis:

$$k = \frac{E(x-\mu)^4}{\sigma^4} \quad (1)$$

where: μ is the mean of x and σ is the standard deviation of x .

The exact kurtosis equation is given in (2):

$$k = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2} \quad (2)$$

Being a measure based on the central moments the above expression for kurtosis is burdened with all the drawbacks of such a measures, the most apparent one is the high sensitivity to the extreme observations. There are other measures of kurtosis based on quantiles and percentiles. The most popular of those measures can be found in [Groenevald, Meeden, 1984; Groenevald, 1998]. However, as this study is mostly focused on extreme values and the quantiles based measures of kurtosis are most suited for symmetrical distributions, the classic measure of kurtosis of Pearson (2) will be utilized. The drawback of using kurtosis is that it allows to draw conclusions only jointly for both of the tails of the distribution and is best suited for symmetrical distribution. This is an important issue as this study aims to compare the differences between left and right tails of the distributions. The following adjustment allows for the use of kurtosis for comparison between both tails: For a given data sample we discard all of the observations above the mean, transform all of the remaining observations symmetrically with respect to the mean and create a perfectly symmetrical data sample by combining the non-discarded observations and their symmetrical transformations. We compute the kurtosis for such a data sample obtaining a measure of the thickness of the left tail. An analogous procedure is then applied to the observations above the mean in order to compute the right-sided kurtosis. The kurtosis of a normal distribution is equal to 3 hence, distributions with kurtosis higher than 3 can be considered fat-tailed.

The other approach utilized in this study is based on the Extreme Value Theory (EVT). Elements of EVT are described in section 3.1 of this article. Further in the study statistical tests are utilized to evaluate the obtained results. Most prominently the Jarque-Bera test of normality [Jarque, Bera, 1987] and the Mann-Whitney U test for the differences between two populations [Mann, Whitney, 1947]. The Jarque-Bera test has a null hypothesis that the data comes from a population which is normally distributed². The test statistic is given in (3):

$$JB = \frac{n}{6} \left(A_d^2 + \frac{1}{4} (k - 3)^2 \right) \quad (3)$$

where: k is the sample kurtosis as defined in (2) and A_d is the sample skewness given in (4)

$$A_d = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^{\frac{3}{2}}} \quad (4)$$

The test statistic (asymptotically) has a chi-squared distribution with two degrees of freedom.

The Mann-Whitney test is a non-parametric test that utilizes ranks in order to test the null hypothesis that two samples come from the same population against an alternative hypothesis that one population consistently has higher values. In order to utilize the Mann-Whitney test one must first sort observations from both samples jointly in an ascending order and assign a rank to each observation. The test statistic is given in (5)

$$U = n_1 n_2 + n_1 \frac{n_1 + 1}{2} - T_1 \quad (5)$$

where: n_1 and n_2 are sample sizes for the first and second sample and T_1 is the sum of all ranks of the observations from the first sample.

It does not matter which sample is treated as the first one and which as the second one. Critical values for the test can be found in many statistic textbooks e.g. [Kanji, 2006, p. 218-221].

3.1. Elements of Extreme Value Theory

Extreme Value Theory is sometimes described as the equivalent of the Central Limit Theorem that applies to the tails of the distributions (rather than their means). The basis of EVT were formulated by Fisher and Tippet [Fisher, Tippet, 1928] and Gnedenko [Gnedenko, 1943], in what became the Fisher-Tippet-Gnedenko theorem: let $x_1, x_2, x_3, \dots, x_m$ be a sequence of m independent and identically distributed random variables with an (unknown) distribution $F(x)$ and let M_n be the maximum of the first $n < m$ observations, then for a sequence of pairs of real numbers $a_n > 0$ b_n with $n \rightarrow \infty$ normalized maxima of the form $(M_n - b_n)/a_n$ converge to the Generalized Extreme Value Distribution (GEV) with the following Cumulative Distribution Function (CDF):

$$H_{\beta\xi}(x) = \begin{cases} e^{-\left(1+\xi\frac{x}{\beta}\right)^{\frac{1}{\xi}}} & \text{for } \xi \neq 0 \\ e^{-e^{-\frac{x}{\beta}}} & \text{for } \xi = 0 \end{cases} \quad (6)$$

² More formally that the skewness and kurtosis, have the same values as the normal distribution

where: $\beta > 0$ and ξ are the parameters of the distribution.

This theorem suggests that no matter what is the initial distribution of random variables, their standardized maxima (defined as above) will asymptotically converge to one of the three distributions: Weibull distribution for $\xi < 0$, what corresponds to thin finite tails, Gumbel distribution for $\xi = 0$, what corresponds to exponentially diminishing tails (as in the case of a normal distribution), or the Fréchet distribution for $\xi > 0$, what corresponds to tails that diminish by a power, this can be described as thick tails (see figure 1).

When it comes to financial applications the second theorem in Extreme Value Theory is more useful, as it applies not to standardized maxima but to the values above a certain (high) threshold. For an unknown distribution function $F(x)$ of random variables x , the distribution of the variable x above a threshold s can be defined as:

$$F_s(y) = P(x - s \leq y | x > s) = \frac{F(y+s) - F(s)}{1 - F(s)} \quad (7)$$

Pickands-Balkema-de Haan theorem [Balkema, de Haan, 1974; Pickands, 1975] states that for a large class of underlying functions F and for $s \rightarrow \infty$ the distribution F_s is well approximated by the Generalized Pareto Distribution (GPD) with the following CDF:

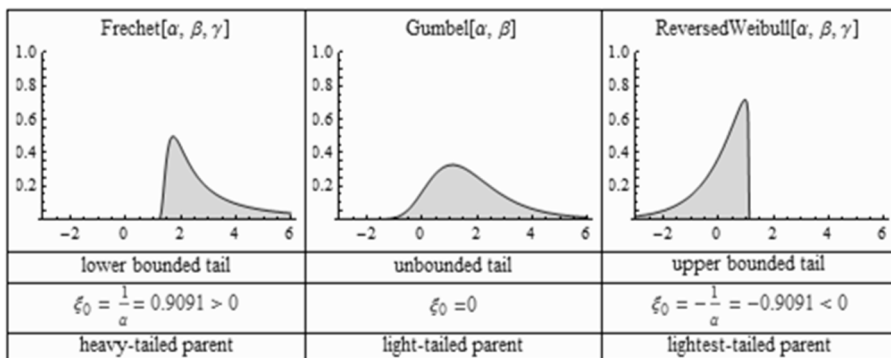
$$G_{\beta\xi}(x) = \begin{cases} 1 - \left(1 + \xi \frac{x}{\beta}\right)^{-\frac{1}{\xi}} & \text{dla } \xi \neq 0 \\ 1 - e^{-\frac{x}{\beta}} & \text{dla } \xi = 0 \end{cases} \quad (8)$$

where: $\beta > 0$ and ξ are parameters

Similarly, as in the case of GEV distribution the ξ parameter determines the thickness of tails. In general: the higher the ξ parameter, the thicker the tail of the distribution. Figure 1. presents sample distributions with varying thickness of right tails according to the corresponding values of ξ

FIGURE 1.

Differences in tails depending on the ξ parameter



Source: own elaboration, based on Wolfram Demonstrations Project, electronic document: [http://demonstrations.wolfram.com/, date of access: 12.03.2014].

The GPD parameters can be estimated with the Maximum Likelihood Method, by maximizing the following log-likelihood function:

$$\mathcal{L}(\xi\beta|k, x_1, \dots, x_n) = -k \times \ln(\beta) - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^k \ln\left(1 + \frac{\xi(x_i - s)}{\beta}\right) \quad (9)$$

where: $\frac{\xi}{\beta}(x_i - s) > -1$, and k is the number of observations exceeding the threshold s .

The parameters can be estimated for both tails of the distribution by using the absolute value of the excess above the threshold $|x_i - s|$.

4. Empirical Results

The return data was normalized with respect to the mean and the standard deviation:

$$z_t = \frac{\mu_t - \bar{\mu}}{\sigma} \quad (10)$$

where: z_t is the standardized return and μ_t is the lognormal return.

For each index the cutoff threshold s was set at the 10th and 90th percentile of the sample data for the left and right tail respectively. The GPD fitting was carried out with the Maximum Likelihood Estimation with the use of Matlab software. The sample kurtosis values were calculated on the unstandardized data according to the equation (2). The following table summarizes the obtained results.

TABLE 2.

Empirical results

Index	WIG	WIG20	mWIG40	MiS80	S&P500	Nikkei225	Bovespa
<i>Country</i>	<i>Poland</i>	<i>Poland</i>	<i>Poland</i>	<i>Poland</i>	<i>U.S.A.</i>	<i>Japan</i>	<i>Brazil</i>
No of obs.	2571	2571	2571	2571	2578	2513	2534
GPD ξ [L]	0.0891	0.0723	0.1197	0.1502	0.2104	0.2542	0.1031
GPD ξ [R]	0.0127	0.0307	0.0119	-0.0425	0.2927	0.2419	0.2545
GPD β [L]	0.7319	0.7068	0.8019	0.8127	0.7075	0.5785	0.6284
GPD β [R]	0.6281	0.6049	0.5973	0.5911	0.5371	0.4169	0.4518
min	-0.0829	-0.0844	-0.0910	-0.0801	-0.0947	-0.1211	-0.1209
max	0.0608	0.0815	0.0512	0.0503	0.1096	0.1323	0.1368
mean	0.0003	0.0002	0.0004	0.0006	0.0002	0.0001	0.0003
std. dev.	0.0131	0.0155	0.0117	0.0114	0.0128	0.0157	0.0181
skewness	-0.4826	-0.2983	-0.9589	-1.1671	-0.3298	-0.5749	-0.0713
kurtosis	6.5055	6.0031	8.5410	8.9674	14.1482	11.2688	8.3368
kurtosis[L]	7.0889	6.3755	9.4174	9.4473	12.1987	11.5957	7.2997
kurtosis[R]	5.2053	5.3562	5.0867	4.6844	16.1960	10.2817	9.3655
jbera	1416.23	1004.26	3682.95	4398.38	13396.69	7297.63	3009.29
p-value (jbera)	~0	~0	~0	~0	~0	~0	~0

Index	AOR	SAX	PX	OSE	OMX Vilnius	OMX Tallinn	OMX Riga
<i>Country</i>	<i>Australia</i>	<i>Slovakia</i>	<i>Czech R.</i>	<i>Norway</i>	<i>Lithuania</i>	<i>Estonia</i>	<i>Latvia</i>
No of obs.	2593	2524	2577	2578	2620	2580	2565
GPD ξ [L]	0.1472	0.2531	0.2340	0.1323	0.2156	0.1447	0.0747
GPD ξ [R]	0.1973	0.0709	0.2840	0.1411	0.2669	0.2473	0.1627
GPD β [L]	0.6558	0.7316	0.6503	0.7744	0.7527	0.7136	0.7311
GPD β [R]	0.5322	0.7177	0.4854	0.5576	0.5620	0.5726	0.6656
min	-0.0855	-0.1481	-0.1619	-0.0971	-0.1194	-0.0705	-0.0786
max	0.0536	0.1188	0.1236	0.0919	0.1100	0.1209	0.1018
mean	0.0002	0.0001	0.0002	0.0005	0.0004	0.0004	0.0001
std. dev.	0.0107	0.0115	0.0153	0.0155	0.0116	0.0118	0.0129
skewness	-0.5594	-1.3751	-0.5521	-0.6485	-0.3605	0.1936	0.1551
kurtosis	8.6984	26.2181	17.2585	9.1263	21.1430	12.4455	9.8399
kurtosis[L]	9.5687	31.0941	17.7981	8.8295	20.7456	9.2590	8.4888
kurtosis[R]	6.6419	17.3933	15.5755	8.3714	20.8907	15.2650	11.0339
jbera	3643.52	57488.57	21960.76	4212.16	35991.04	9606.91	5010.39
p-value (jbera)	~0	~0	~0	~0	~0	~0	~0

Index	OMX Helsinki	IBEX	FTSE250	DAX	CAC40	BUX
<i>Country</i>	<i>Finland</i>	<i>Spain</i>	<i>U.K.</i>	<i>Germany</i>	<i>France</i>	<i>Hungary</i>
No of obs.	2579	2609	2590	2612	2627	2567
GPD ξ [L]	0.0355	0.0263	-0.0557	0.1093	0.0643	0.1742
GPD ξ [R]	0.1860	0.2253	0.0992	0.2447	0.2732	0.1828
GPD β [L]	0.7380	0.7756	0.8816	0.6882	0.7520	0.5775
GPD β [R]	0.5788	0.5250	0.6029	0.5045	0.4846	0.5380
min	-0.0923	-0.0959	-0.0673	-0.0743	-0.0947	-0.1265
max	0.0885	0.1348	0.0746	0.1080	0.1059	0.1318
mean	7.84E-05	0.0001	0.0004	0.0003	7.9E-05	0.0002
std. dev.	0.0145	0.0150	0.0117	0.0138	0.0143	0.0168
skewness	-0.1341	0.1185	-0.3399	0.0186	0.0418	-0.0863
kurtosis	7.3108	10.0971	6.7275	10.0149	10.0437	9.3842
kurtosis[L]	6.6818	7.1270	6.2031	7.1391	7.7945	9.2250
kurtosis[R]	7.9712	13.4992	6.9358	13.3189	12.6234	9.4095
jbera	2004.60	5481.73	1549.30	5355.72	5431.32	4362.59
p-value (jbera)	~0	~0	~0	~0	~0	~0

Source: own elaboration

Apart from the GPD parameters and kurtosis, skewness and Jarque-Bera statistic were also calculated. The instances where the ξ parameter or the kurtosis indicates that the left tail is fatter are highlighted with bold font. In most cases both of the used methods indicate that the same tail is the heavier one. This was not true for three indices the Australian AOR, the Czech PX and the Norwegian OSE in which case the ξ parameter

indicated a fatter right tail and the kurtosis a fatter left tail. Based on the GPD fitting criterion there were 14 cases in which the right tail was heavier (S&P 500, Bovespa, AOR, PX, OSE, OMX Vilnius, OMX Tallinn, OMX Riga, OMX Helsinki, IBEX, FTSE 250 DAX, CAC40 and BUX), kurtosis indicated that eleven of the studied indices have a fatter right tail. Those results do not confirm the stylized fact that the left tail of a stock market return distribution is heavier than the right one. In fact both methods indicated the opposite. There were two instances in which the ξ parameter although close to zero was actually negative what can be interpreted as an indication of thin tails. Those were the left tail of the London Stock Exchange FTSE 250 Index and the right tail of the Warsaw Stock Exchange MiS80 index. There were five instances of positive in-sample skewness, those were: OMX Tallinn, OMX Riga, IBEX, DAX, and CAC 40 indices. In all cases the Jarque-Bera test strongly indicates that the data is not normally distributed.

In order to statistically test the difference between the two tails the differences between the left and right tail ξ parameters were calculated for each index:

$$\xi_{\text{dif},i} = \xi_{L,i} - \xi_{R,i} \quad (11)$$

The results of the Jarque-Bera and Student t tests on the series of the differences of the ξ parameters ($\xi_{\text{dif},i}$) and additionally the results of the Mann-Whitney U-test for the comparison between the series of the right sided ($\xi_{R,i}$), and left sided ($\xi_{L,i}$) parameters are summarized in the table 3.

TABLE 3.

Statistical tests

Test	Statistic	p - value	No basis to reject the null hypothesis (90%)
Jarque-Bera	1.221	0.311	H0 - data normal
Student t	-1.587	0.129	H0 - mean difference = 0
Mann-Whitney U	353	0.126	H0 - both populations equal

Source: own elaboration.

The Jarque-Bera test indicates that the difference between the left sided parameter and the right sided parameter is normally distributed and the student t test suggests that the mean of that distribution is equal to zero with no sufficient evidence to reject the null hypothesis at 10% significance level. The Mann-Whitney test also suggests that both populations are equal (again no basis to reject the null hypothesis). This can be interpreted that the difference between the thickness of the right and left tail is a random white-noise-like process without a preference for any of the tails.

5. Conclusions

The obtained results do not confirm the stylized fact that the left tail of the distribution of stock market returns is thicker than the right tail. The statistical tests suggest that it

may be equally likely for either of the tails to be thicker. The kurtosis-based measure indicated that the left tail was thicker in more than half of the studied cases and the ξ parameter of the GPD indicated that only 6 out of 20 indices exhibited a fatter left tail. With short selling and other more complex investment strategies becoming more widely used the classic association of the left tail with the probability of an investment loss becomes outdated. In this situation a fatter right tail cannot be interpreted simply as a bigger probability of an extreme gain. It is possible that with time left and right tails of stock market return distributions will become more similar. The lack of statistical significance of the differences between the tails of stock market returns is in line with the study by Jondeau and Rockinger [Jondeau, Rockinger, 2003], however, the results obtained in this paper show even less support for the hypothesis that the left tail is heavier.

The results do not suggest however that the distributions of stock market returns are symmetrical, as the big majority of examined indices (15 out of 20) exhibited negative skewness. The Jarque-Bera tests strongly indicate that daily stock market returns are not normally distributed, and the measures of the thickness of tails show that almost all of them come from thick-tailed distributions with Fréchet-type tails. A further study is needed to test whether or not those characteristics (especially lack of significant differences between tails) change with time and market circumstances. This is problematic as methods based on Extreme Value Theory in general require very long time series.

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