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On the Calculus of Binary Arithmetics. Part II

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Summary. In this paper, we introduce binary arithmetic and its related operations. We include some theorems concerning logical operators.

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The terminology and notation used in this paper are introduced in the following articles: [4], [3], [2], and [1].

In this paper x, y, z denote boolean sets.

Next we state a number of propositions:

(1) $\mathit{true} \Rightarrow x = x$.

(2) $\mathit{false} \Rightarrow x = \mathit{true}$.

(3) $x \Rightarrow x = \mathit{true}$ and $\neg(x \Rightarrow x) = \mathit{false}$.

(4) $\neg(x \Rightarrow y) = x \wedge \neg y$.

(5) $x \Rightarrow \neg x = \neg x$ and $\neg(x \Rightarrow \neg x) = x$.

(6) $\neg x \Rightarrow x = x$.

(7) $\mathit{true} \Leftrightarrow x = x$.

(8) $\mathit{false} \Leftrightarrow x = \neg x$.

(9) $x \Leftrightarrow x = \mathit{true}$ and $\neg(x \Leftrightarrow x) = \mathit{false}$.

(10) $\neg x \Leftrightarrow x = \mathit{false}$.

(11) $x \wedge (y \Leftrightarrow z) = x \wedge (\neg y \vee z) \wedge (\neg z \vee y)$.

(12) $x \wedge (y \mathbin{'\mathrm{nand}'} z) = x \wedge \neg y \vee x \wedge \neg z$.

(13) $x \wedge (y \mathbin{'\mathrm{nor}'} z) = x \wedge \neg y \wedge \neg z$.

(14) $x \wedge (x \wedge y) = x \wedge y$.

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(15) $x \wedge (x \vee y) = x \vee x \wedge y$.

(16) $x \wedge (x \oplus y) = x \wedge \neg y$.

(17) $x \wedge (x \Rightarrow y) = x \wedge y$.

(18) $x \wedge (x \Leftrightarrow y) = x \wedge y$.

(19) $x \wedge (x\ 'nand'\ y) = x \wedge \neg y$.

(20) $x \wedge (x\ 'nor'\ y) = \mathit{false}$.

(21) $x \vee (y \oplus z) = x \vee \neg y \wedge z \vee y \wedge \neg z$.

(22) $x \vee (y \Leftrightarrow z) = (x \vee \neg y \vee z) \wedge (x \vee \neg z \vee y)$.

(23) $x \vee (y\ 'nand'\ z) = x \vee \neg y \vee \neg z$.

(24) $x \vee (y\ 'nor'\ z) = (x \vee \neg y) \wedge (x \vee \neg z)$ and $x \vee (y\ 'nor'\ z) = (y \Rightarrow x) \wedge (z \Rightarrow x)$.

(25) $x \vee (x \vee y) = x \vee y$.

(26) $x \vee (x \Rightarrow y) = \mathit{true}$.

(27) $x \vee (x \Leftrightarrow y) = y \Rightarrow x$.

(28) $x \vee (x\ 'nand'\ y) = \mathit{true}$.

(29) $x \vee (x\ 'nor'\ y) = y \Rightarrow x$.

(30) $x \Rightarrow y \oplus z = \neg x \vee \neg y \wedge z \vee y \wedge \neg z$.

(31) $x \Rightarrow y \Leftrightarrow z = (\neg x \vee \neg y \vee z) \wedge (\neg x \vee y \vee \neg z)$.

(32) $x \Rightarrow y\ 'nand'\ z = \neg x \vee \neg y \vee \neg z$.

(33) $x \Rightarrow y\ 'nor'\ z = (\neg x \vee \neg y) \wedge (\neg x \vee \neg z)$ and $x \Rightarrow y\ 'nor'\ z = (x \Rightarrow \neg y) \wedge (x \Rightarrow \neg z)$.

(34) $x \Rightarrow x \wedge y = x \Rightarrow y$.

(35) $x \Rightarrow x \vee y = \mathit{true}$.

(36) $x \Rightarrow x \oplus y = \neg x \vee \neg y$.

(37) $x \Rightarrow x \Rightarrow y = x \Rightarrow y$.

(38) $x \Rightarrow x \Leftrightarrow y = x \Rightarrow y$ and $x \Rightarrow x \Leftrightarrow y = x \Rightarrow x \Rightarrow y$.

(39) $x \Rightarrow x\ 'nand'\ y = \neg(x \wedge y)$.

(40) $x \Rightarrow x\ 'nor'\ y = \neg x$.

(41) $x\ 'nand'\ (y \Rightarrow z) = (\neg x \vee y) \wedge (\neg x \vee \neg z)$ and $x\ 'nand'\ (y \Rightarrow z) = (x \Rightarrow y) \wedge (x \Rightarrow \neg z)$.

(42) $x\ 'nand'\ (y \Leftrightarrow z) = \neg(x \wedge (\neg y \vee z) \wedge (\neg z \vee y))$.

(43) $x\ 'nand'\ (y\ 'nand'\ z) = (\neg x \vee y) \wedge (\neg x \vee z)$ and $x\ 'nand'\ (y\ 'nand'\ z) = (x \Rightarrow y) \wedge (x \Rightarrow z)$.

(44) $x\ 'nand'\ (y\ 'nor'\ z) = \neg x \vee y \vee z$.

(45) $x\ 'nand'\ x \wedge y = \neg(x \wedge y)$.

(46) $x\ 'nand'\ (x \oplus y) = x \Rightarrow y$.

(47) $x\ 'nand'\ (x \Rightarrow y) = \neg(x \wedge y)$.

(48) $x\ 'nand'\ (x \Leftrightarrow y) = \neg(x \wedge y)$.

(49) $x\ {}'\mathrm{nand}'\ (x\ {}'\mathrm{nand}'\ y) = x \Rightarrow y.$
(50) $x\ {}'\mathrm{nand}'\ (x\ {}'\mathrm{nor}'\ y) = \mathit{true}.$
(51) $x\ {}'\mathrm{nor}'\ (y \oplus z) = \neg(x \vee \neg y \wedge z \vee y \wedge \neg z).$
(52) $x\ {}'\mathrm{nor}'\ (y \Leftrightarrow z) = \neg((x \vee \neg y \vee z) \wedge (x \vee \neg z \vee y)).$
(53) $x\ {}'\mathrm{nor}'\ (y\ {}'\mathrm{nand}'\ z) = \neg x \wedge y \wedge z.$
(54) $x\ {}'\mathrm{nor}'\ (y\ {}'\mathrm{nor}'\ z) = \neg x \wedge y \vee \neg x \wedge z.$
(55) $x\ {}'\mathrm{nor}'\ x \wedge y = \neg x.$
(56) $x\ {}'\mathrm{nor}'\ (x \vee y) = \neg x \wedge \neg y.$
(57) $x\ {}'\mathrm{nor}'\ (x \oplus y) = \neg x \wedge \neg y.$
(58) $x\ {}'\mathrm{nor}'\ (x \Rightarrow y) = \mathit{false}.$
(59) $x\ {}'\mathrm{nor}'\ (x \Leftrightarrow y) = \neg x \wedge y.$
(60) $x\ {}'\mathrm{nor}'\ (x\ {}'\mathrm{nand}'\ y) = \mathit{false}.$
(61) $x\ {}'\mathrm{nor}'\ (x\ {}'\mathrm{nor}'\ y) = \neg x \wedge y.$
(62) $x \oplus y \wedge z = (x \vee y \wedge z) \wedge (\neg x \vee \neg(y \wedge z)).$
(63) $x \oplus x \wedge y = x \wedge \neg y.$
(64) $x \oplus (x \vee y) = \neg x \wedge y.$
(65) $\neg x \wedge (x \oplus y) = \neg x \wedge y.$
(66) $x \wedge \neg(x \oplus y) = x \wedge y.$
(67) $x \oplus (x \oplus y) = y.$
(68) $x \wedge \neg(x \Rightarrow y) = x \wedge \neg y.$
(69) $x \oplus (x \Rightarrow y) = \neg x \vee \neg y.$
(70) $\neg x \wedge (x \Leftrightarrow y) = \neg x \wedge \neg y.$
(71) $x \wedge \neg(x \Leftrightarrow y) = x \wedge \neg y.$
(72) $x \oplus (x \Leftrightarrow y) = \neg y.$
(73) $x \oplus (x\ {}'\mathrm{nand}'\ y) = x \Rightarrow y.$
(74) $x \oplus (x\ {}'\mathrm{nor}'\ y) = y \Rightarrow x.$
(75) $\neg x \wedge (x \Rightarrow y) = \neg x \vee \neg x \wedge y.$
(76) $\neg x \wedge (y \Leftrightarrow z) = \neg x \wedge (\neg y \vee z) \wedge (\neg z \vee y).$
(77) $\neg x \wedge (x \Leftrightarrow y) = \neg x \wedge \neg y \wedge (\neg x \vee y).$
(78) $\neg x \wedge (x\ {}'\mathrm{nand}'\ y) = \neg x \vee \neg x \wedge \neg y.$
(79) $\neg x \wedge (x\ {}'\mathrm{nor}'\ y) = \neg x \wedge \neg y.$
(80) $\neg x \vee (x \Rightarrow y) = \neg x \vee y.$
(81) $\neg x \vee (x \Leftrightarrow y) = \neg x \vee y.$
(82) $\neg x \vee (x\ {}'\mathrm{nand}'\ y) = \neg x \vee \neg y.$
(83) $\neg x \oplus (x \Rightarrow y) = x \wedge y.$
(84) $\neg x \oplus (y \Rightarrow x) = x \wedge (x \vee \neg y) \vee \neg x \wedge y.$
(85) $\neg(x \Rightarrow y) = x \wedge \neg y.$

(86) $\neg(x \Leftrightarrow y) = x \wedge \neg y \vee y \wedge \neg x$.

(87) $\neg x \oplus (x \Leftrightarrow y) = y$.

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