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Homeomorphisms of Jordan Curves

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Summary. In this paper we prove that simple closed curves can be homeomorphically framed into a given rectangle. We also show that homeomorphisms preserve the Jordan property.

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The notation and terminology used in this paper are introduced in the following articles: [20], [21], [1], [3], [22], [4], [5], [19], [10], [18], [7], [17], [11], [2], [8], [9], [16], [13], [14], [15], [6], [23], and [12].

In this paper p_1, p_2 are points of $\mathcal{E}^2_{\mathrm{T}}$, C is a simple closed curve, and P is a subset of $\mathcal{E}^2_{\mathrm{T}}$.

Let n be a natural number, let A be a subset of $\mathcal{E}^n_{\mathrm{T}}$, and let a, b be points of $\mathcal{E}^n_{\mathrm{T}}$. We say that a and b realize maximal distance in A if and only if:

(Def. 1) $a \in A$ and $b \in A$ and for all points x, y of $\mathcal{E}^n_{\mathrm{T}}$ such that $x \in A$ and $y \in A$ holds $\rho(a, b) \geq \rho(x, y)$.

Next we state the proposition

(1) There exist p_1, p_2 such that p_1 and p_2 realize maximal distance in C.

Let M be a non empty metric structure and let f be a map from M_{top} into M_{top}. We say that f is isometric if and only if:

(Def. 2) There exists an isometric map g from M into M such that $g = f$.

Let M be a non empty metric structure. Note that there exists a map from M_{top} into M_{top} which is isometric.

Let M be a non empty metric space. Observe that every map from M_{top} into M_{top} which is isometric is also continuous.

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Let M be a non empty metric space. Note that every map from M_{top} into M_{top} which is isometric is also homeomorphism.

Let a be a real number. The functor Rotate a yields a map from $\mathcal{E}^2_{\mathrm{T}}$ into $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 3) For every point p of $\mathcal{E}^2_{\mathrm{T}}$ holds $(\mathrm{Rotate}\, a)(p) = [\Re(p_{\mathbf{1}} + p_{\mathbf{2}} \cdot i \circlearrowleft a), \Im(p_{\mathbf{1}} + p_{\mathbf{2}} \cdot i \circlearrowleft a)]$, where $a = [r_1, 0]$ and $r_1 = -1$.

The following propositions are true:

(2) Let a be a real number. Suppose $0 \leq a$ and $a < 2 \cdot \pi$. Let f be a map from $(\mathcal{E}^2)_{\mathrm{top}}$ into $(\mathcal{E}^2)_{\mathrm{top}}$. If $f = \mathrm{Rotate}\, a$, then f is isometric, where $a = [r_1, 0]$ and $r_1 = -1$.

(3) Let A, B, D be real numbers. Suppose p_1 and p_2 realize maximal distance in P. Then $(\mathrm{AffineMap}(A, B, A, D))(p_1)$ and $(\mathrm{AffineMap}(A, B, A, D))(p_2)$ realize maximal distance in $(\mathrm{AffineMap}(A, B, A, D))^\circ P$.

(4) Let A be a real number. Suppose $0 \leq A$ and $A < 2 \cdot \pi$ and p_1 and p_2 realize maximal distance in P. Then $(\mathrm{Rotate}\, A)(p_1)$ and $(\mathrm{Rotate}\, A)(p_2)$ realize maximal distance in $(\mathrm{Rotate}\, A)^\circ P$.

(5) For every complex number z and for every real number r holds $z \circlearrowleft -r = z \circlearrowleft 2 \cdot \pi - r$.

(6) For every real number r holds $\mathrm{Rotate}(-r) = \mathrm{Rotate}(2 \cdot \pi - r)$.

(7) There exists a homeomorphism f of $\mathcal{E}^2_{\mathrm{T}}$ such that $[-1, 0]$ and $[1, 0]$ realize maximal distance in $f^\circ C$.

Let T_1, T_2 be topological structures and let f be a map from T_1 into T_2. We say that f is closed if and only if:

(Def. 4) For every subset A of T_1 such that A is closed holds $f^\circ A$ is closed.

One can prove the following propositions:

(8) Let X, Y be non empty topological spaces and f be a continuous map from X into Y. Suppose f is one-to-one and onto. Then f is a homeomorphism if and only if f is closed.

(9) For every set X and for every subset A of X holds $A^{\mathrm{c}} = \emptyset$ iff $A = X$.

(10) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A be a subset of T_1. If A is connected, then $f^\circ A$ is connected.

(11) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A be a subset of T_1. If A is a component of T_1, then $f^\circ A$ is a component of T_2.

(12) Let T_1, T_2 be non empty topological spaces, f be a map from T_1 into T_2, and A be a subset of T_1. Then $f \restriction A$ is a map from $T_1 \restriction A$ into $T_2 \restriction f^\circ A$.

(13) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into

T_2. Suppose f is continuous. Let A be a subset of T_1 and g be a map from $T_1 \restriction A$ into $T_2 \restriction f^\circ A$. If $g = f \restriction A$, then g is continuous.

(14) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A be a subset of T_1 and g be a map from $T_1 \restriction A$ into $T_2 \restriction f^\circ A$. If $g = f \restriction A$, then g is a homeomorphism.

(15) Let T_1, T_2 be non empty topological spaces and f be a map from T_1 into T_2. Suppose f is a homeomorphism. Let A, B be subsets of T_1. If A is a component of B, then $f^\circ A$ is a component of $f^\circ B$.

(16) For every subset S of $\mathcal{E}^2_{\mathrm{T}}$ and for every homeomorphism f of $\mathcal{E}^2_{\mathrm{T}}$ such that S is Jordan holds $f^\circ S$ is Jordan.

References

[1] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.
[2] Leszek Borys. Paracompact and metrizable spaces. *Formalized Mathematics*, 2(**4**):481–485, 1991.
[3] Czesław Byliński. The complex numbers. *Formalized Mathematics*, 1(**3**):507–513, 1990.
[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[5] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[6] Wenpai Chang, Yatsuka Nakamura, and Piotr Rudnicki. Inner products and angles of complex numbers. *Formalized Mathematics*, 11(**3**):275–280, 2003.
[7] Agata Darmochwał. Families of subsets, subspaces and mappings in topological spaces. *Formalized Mathematics*, 1(**2**):257–261, 1990.
[8] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.
[9] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Simple closed curves. *Formalized Mathematics*, 2(**5**):663–664, 1991.
[10] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[11] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Formalized Mathematics*, 1(**3**):607–610, 1990.
[12] Artur Korniłowicz. The definition and basic properties of topological groups. *Formalized Mathematics*, 7(**2**):217–225, 1998.
[13] Artur Korniłowicz. Properties of left and right components. *Formalized Mathematics*, 8(**1**):163–168, 1999.
[14] Robert Milewski. Real linear-metric space and isometric functions. *Formalized Mathematics*, 7(**2**):273–277, 1998.
[15] Yatsuka Nakamura. On Outside Fashoda Meet Theorem. *Formalized Mathematics*, 9(**4**):697–704, 2001.
[16] Yatsuka Nakamura and Jarosław Kotowicz. The Jordan's property for certain subsets of the plane. *Formalized Mathematics*, 3(**2**):137–142, 1992.
[17] Beata Padlewska. Connected spaces. *Formalized Mathematics*, 1(**1**):239–244, 1990.
[18] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[19] Andrzej Trybulec. Subsets of complex numbers. *To appear in Formalized Mathematics*.
[20] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[21] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[22] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

[23] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

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