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Concatenation of Finite Sequences Reducing Overlapping Part and an Argument of Separators of Sequential Files

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Summary. For two finite sequences, we present a notion of their concatenation, reducing overlapping part of the tail of the former and the head of the latter. At the same time, we also give a notion of common part of two finite sequences, which relates to the concatenation given here. A finite sequence is separated by another finite sequence (separator). We examined the condition that a separator separates uniquely any finite sequence. This will become a model of a separator of sequential files.

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The terminology and notation used here are introduced in the following articles: [14], [15], [9], [1], [12], [16], [3], [10], [2], [4], [5], [8], [13], [7], [11], and [6].

The following propositions are true:

(1) For every set D and for every finite sequence f of elements of D holds $f \restriction 0 = \emptyset$.

(2) For every set D and for every finite sequence f of elements of D holds $f_{\restriction 0} = f$.

Let D be a set and let f, g be finite sequences of elements of D. Then $f \frown g$ is a finite sequence of elements of D.

Next we state three propositions:

(3) For every non empty set D and for all finite sequences f, g of elements of D such that $\operatorname{len} f \geqslant 1$ holds $\operatorname{mid}(f \frown g, 1, \operatorname{len} f) = f$.

(4) Let D be a set, f be a finite sequence of elements of D, and i be a natural number. If $i \geqslant \operatorname{len} f$, then $f_{\restriction i} = \varepsilon_D$.

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(5) For every non empty set D and for all natural numbers k_1, k_2 holds $\mathrm{mid}(\varepsilon_D, k_1, k_2) = \varepsilon_D$.

Let D be a set, let f be a finite sequence of elements of D, and let k_1, k_2 be natural numbers. The functor $\mathrm{smid}(f, k_1, k_2)$ yields a finite sequence of elements of D and is defined as follows:

(Def. 1) $\mathrm{smid}(f, k_1, k_2) = f_{\lfloor k_1 -' 1} \restriction ((k_2 + 1) -' k_1)$.

One can prove the following propositions:

(6) Let D be a non empty set, f be a finite sequence of elements of D, and k_1, k_2 be natural numbers. If $k_1 \leqslant k_2$, then $\mathrm{smid}(f, k_1, k_2) = \mathrm{mid}(f, k_1, k_2)$.

(7) Let D be a non empty set, f be a finite sequence of elements of D, and k_2 be a natural number. Then $\mathrm{smid}(f, 1, k_2) = f \restriction k_2$.

(8) Let D be a non empty set, f be a finite sequence of elements of D, and k_2 be a natural number. If $\mathrm{len}\, f \leqslant k_2$, then $\mathrm{smid}(f, 1, k_2) = f$.

(9) Let D be a set, f be a finite sequence of elements of D, and k_1, k_2 be natural numbers. If $k_1 > k_2$, then $\mathrm{smid}(f, k_1, k_2) = \emptyset$ and $\mathrm{smid}(f, k_1, k_2) = \varepsilon_D$.

(10) For every set D and for every finite sequence f of elements of D and for every natural number k_2 holds $\mathrm{smid}(f, 0, k_2) = \mathrm{smid}(f, 1, k_2 + 1)$.

(11) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{smid}(f \frown g, \mathrm{len}\, f + 1, \mathrm{len}\, f + \mathrm{len}\, g) = g$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor $\mathrm{ovlpart}(f, g)$ yielding a finite sequence of elements of D is defined by the conditions (Def. 2).

(Def. 2)(i) $\mathrm{len}\, \mathrm{ovlpart}(f, g) \leqslant \mathrm{len}\, g$,

(ii) $\mathrm{ovlpart}(f, g) = \mathrm{smid}(g, 1, \mathrm{len}\, \mathrm{ovlpart}(f, g))$,

(iii) $\mathrm{ovlpart}(f, g) = \mathrm{smid}(f, (\mathrm{len}\, f -' \mathrm{len}\, \mathrm{ovlpart}(f, g)) + 1, \mathrm{len}\, f)$, and

(iv) for every natural number j such that $j \leqslant \mathrm{len}\, g$ and $\mathrm{smid}(g, 1, j) = \mathrm{smid}(f, (\mathrm{len}\, f -' j) + 1, \mathrm{len}\, f)$ holds $j \leqslant \mathrm{len}\, \mathrm{ovlpart}(f, g)$.

Next we state the proposition

(12) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{len}\, \mathrm{ovlpart}(f, g) \leqslant \mathrm{len}\, f$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor $\mathrm{ovlcon}(f, g)$ yielding a finite sequence of elements of D is defined as follows:

(Def. 3) $\mathrm{ovlcon}(f, g) = f \frown (g_{\lfloor \mathrm{len}\, \mathrm{ovlpart}(f,g)})$.

One can prove the following proposition

(13) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{ovlcon}(f, g) = (f \restriction (\mathrm{len}\, f -' \mathrm{len}\, \mathrm{ovlpart}(f, g))) \frown g$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor ovlldiff(f, g) yields a finite sequence of elements of D and is defined as follows:

(Def. 4) ovlldiff$(f, g) = f\restriction(\operatorname{len} f -' \operatorname{len} \operatorname{ovlpart}(f, g))$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor ovlrdiff(f, g) yields a finite sequence of elements of D and is defined by:

(Def. 5) ovlrdiff$(f, g) = g_{\restriction \operatorname{len} \operatorname{ovlpart}(f,g)}$.

One can prove the following propositions:

(14) Let D be a non empty set and f, g be finite sequences of elements of D. Then ovlcon$(f, g) = (\text{ovlldiff}(f, g)) \frown \text{ovlpart}(f, g) \frown \text{ovlrdiff}(f, g)$ and ovlcon$(f, g) = (\text{ovlldiff}(f, g)) \frown ((\text{ovlpart}(f, g)) \frown \text{ovlrdiff}(f, g))$.

(15) Let D be a non empty set and f be a finite sequence of elements of D. Then ovlcon$(f, f) = f$ and ovlpart$(f, f) = f$ and ovlldiff$(f, f) = \emptyset$ and ovlrdiff$(f, f) = \emptyset$.

(16) For every non empty set D and for all finite sequences f, g of elements of D holds ovlpart$(f \frown g, g) = g$ and ovlpart$(f, f \frown g) = f$.

(17) Let D be a non empty set and f, g be finite sequences of elements of D. Then $\operatorname{len} \operatorname{ovlcon}(f, g) = (\operatorname{len} f + \operatorname{len} g) - \operatorname{len} \operatorname{ovlpart}(f, g)$ and $\operatorname{len} \operatorname{ovlcon}(f, g) = (\operatorname{len} f + \operatorname{len} g) -' \operatorname{len} \operatorname{ovlpart}(f, g)$ and $\operatorname{len} \operatorname{ovlcon}(f, g) = \operatorname{len} f + (\operatorname{len} g -' \operatorname{len} \operatorname{ovlpart}(f, g))$.

(18) For every non empty set D and for all finite sequences f, g of elements of D holds $\operatorname{len} \operatorname{ovlpart}(f, g) \leqslant \operatorname{len} f$ and $\operatorname{len} \operatorname{ovlpart}(f, g) \leqslant \operatorname{len} g$.

Let D be a non empty set and let C_1 be a finite sequence of elements of D. We say that C_1 separates uniquely if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let f be a finite sequence of elements of D and i, j be natural numbers. Suppose $1 \leqslant i$ and $i < j$ and $(j + \operatorname{len} C_1) -' 1 \leqslant \operatorname{len} f$ and $\operatorname{smid}(f, i, (i + \operatorname{len} C_1) -' 1) = \operatorname{smid}(f, j, (j + \operatorname{len} C_1) -' 1)$ and $\operatorname{smid}(f, i, (i + \operatorname{len} C_1) -' 1) = C_1$. Then $j -' i \geqslant \operatorname{len} C_1$.

The following proposition is true

(19) Let D be a non empty set and C_1 be a finite sequence of elements of D. Then C_1 separates uniquely if and only if $\operatorname{len} \operatorname{ovlpart}((C_1)_{\restriction 1}, C_1) = 0$.

Let D be a non empty set, let f, g be finite sequences of elements of D, and let n be a natural number. We say that g is a substring of f if and only if:

(Def. 7) If $\operatorname{len} g > 0$, then there exists a natural number i such that $n \leqslant i$ and $i \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, i, (i -' 1) + \operatorname{len} g) = g$.

We now state four propositions:

(20) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. If $\operatorname{len} g = 0$, then g is a substring of f.

(21) Let D be a non empty set, f, g be finite sequences of elements of D, and n, m be natural numbers. If $m \geqslant n$ and g is a substring of f, then g is a substring of f.

(22) For every non empty set D and for every finite sequence f of elements of D such that $1 \leqslant \operatorname{len} f$ holds f is a substring of f.

(23) Let D be a non empty set and f, g be finite sequences of elements of D. If g is a substring of f, then g is a substring of f.

Let D be a non empty set and let f, g be finite sequences of elements of D. We say that g is a preposition of f if and only if:

(Def. 8) If $\operatorname{len} g > 0$, then $1 \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, 1, \operatorname{len} g) = g$.

One can prove the following four propositions:

(24) Let D be a non empty set and f, g be finite sequences of elements of D. If $\operatorname{len} g = 0$, then g is a preposition of f.

(25) For every non empty set D holds every finite sequence f of elements of D is a preposition of f.

(26) Let D be a non empty set and f, g be finite sequences of elements of D. If g is a preposition of f, then $\operatorname{len} g \leqslant \operatorname{len} f$.

(27) Let D be a non empty set and f, g be finite sequences of elements of D. If $\operatorname{len} g > 0$ and g is a preposition of f, then $g(1) = f(1)$.

Let D be a non empty set and let f, g be finite sequences of elements of D. We say that g is a postposition of f if and only if:

(Def. 9) $\operatorname{Rev}(g)$ is a preposition of $\operatorname{Rev}(f)$.

Next we state several propositions:

(28) Let D be a non empty set and f, g be finite sequences of elements of D. If $\operatorname{len} g = 0$, then g is a postposition of f.

(29) Let D be a non empty set and f, g be finite sequences of elements of D. If g is a postposition of f, then $\operatorname{len} g \leqslant \operatorname{len} f$.

(30) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. Suppose g is a postposition of f. If $\operatorname{len} g > 0$, then $\operatorname{len} g \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, (\operatorname{len} f + 1) -' \operatorname{len} g, \operatorname{len} f) = g$.

(31) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number such that if $\operatorname{len} g > 0$, then $\operatorname{len} g \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, (\operatorname{len} f + 1) -' \operatorname{len} g, \operatorname{len} f) = g$. Then g is a postposition of f.

(32) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. If $\operatorname{len} g = 0$, then g is a preposition of f.

(33) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. If $1 \leqslant \operatorname{len} f$ and g is a preposition of f, then g is

a substring of f.

(34) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. Suppose g is not a substring of f. Let i be a natural number. If $n \leqslant i$ and $0 < i$, then $\mathrm{mid}(f, i, (i -' 1) + \mathrm{len}\, g) \neq g$.

Let D be a non empty set, let f, g be finite sequences of elements of D, and let n be a natural number. The functor $\mathrm{instr}(n, f)$ yielding a natural number is defined by the conditions (Def. 10).

(Def. 10)(i) If $\mathrm{instr}(n, f) \neq 0$, then $n \leqslant \mathrm{instr}(n, f)$ and g is a preposition of $f_{\restriction \mathrm{instr}(n,f)-'1}$ and for every natural number j such that $j \geqslant n$ and $j > 0$ and g is a preposition of $f_{\restriction j-'1}$ holds $j \geqslant \mathrm{instr}(n, f)$, and

(ii) if $\mathrm{instr}(n, f) = 0$, then g is not a substring of f.

Let D be a non empty set and let f, C_1 be finite sequences of elements of D. The functor $\mathrm{addcr}(f, C_1)$ yields a finite sequence of elements of D and is defined by:

(Def. 11) $\mathrm{addcr}(f, C_1) = \mathrm{ovlcon}(f, C_1)$.

Let D be a non empty set and let r, C_1 be finite sequences of elements of D. We say that r is terminated by C_1 if and only if:

(Def. 12) If $\mathrm{len}\, C_1 > 0$, then $\mathrm{len}\, r \geqslant \mathrm{len}\, C_1$ and $\mathrm{instr}(1, r) = (\mathrm{len}\, r + 1) -' \mathrm{len}\, C_1$.

The following proposition is true

(35) For every non empty set D holds every finite sequence f of elements of D is terminated by f.

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