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Several Differentiable Formulas of Special Functions

Yan Zhang
Qingdao University of Science and Technology
China

Xiquan Liang
Qingdao University of Science and Technology
China

Summary. In this article, we give several differentiable formulas of special functions. There are some specific composite functions consisting of rational functions, irrational functions, trigonometric functions, exponential functions or logarithmic functions.

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The notation and terminology used in this paper have been introduced in the following articles: [13], [15], [16], [1], [4], [10], [12], [3], [6], [9], [7], [8], [11], [17], [5], [14], and [2].

For simplicity, we follow the rules: x, a, b, c denote real numbers, n denotes a natural number, Z denotes an open subset of $\mathbb{R}$, and f, f_1, f_2 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) Suppose $Z \subseteq \operatorname{dom}(\log_{-}(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $\log_{-}(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{-}(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{a+x}$.

(2) Suppose $Z \subseteq \operatorname{dom}(\log_{-}(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = x - a$ and $f(x) > 0$. Then $\log_{-}(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{-}(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{x-a}$.

(3) Suppose $Z \subseteq \operatorname{dom}(-\log_{-}(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $-\log_{-}(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log_{-}(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{a-x}$.

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(4) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z - a\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$. Then $\operatorname{id}_Z - a\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z - a\, f)'_{\restriction Z}(x) = \frac{x}{a+x}$.

(5) Suppose $Z \subseteq \operatorname{dom}((2 \cdot a)\, f - \operatorname{id}_Z)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$. Then $(2 \cdot a)\, f - \operatorname{id}_Z$ is differentiable on Z and for every x such that $x \in Z$ holds $((2 \cdot a)\, f - \operatorname{id}_Z)'_{\restriction Z}(x) = \frac{a-x}{a+x}$.

(6) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z - (2 \cdot a)\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + a$ and $f_1(x) > 0$. Then $\operatorname{id}_Z - (2 \cdot a)\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z - (2 \cdot a)\, f)'_{\restriction Z}(x) = \frac{x-a}{x+a}$.

(7) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z + (2 \cdot a)\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$. Then $\operatorname{id}_Z + (2 \cdot a)\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z + (2 \cdot a)\, f)'_{\restriction Z}(x) = \frac{x+a}{x-a}$.

(8) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z + (a - b)\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + b$ and $f_1(x) > 0$. Then $\operatorname{id}_Z + (a - b)\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z + (a - b)\, f)'_{\restriction Z}(x) = \frac{x+a}{x+b}$.

(9) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z + (a + b)\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - b$ and $f_1(x) > 0$. Then $\operatorname{id}_Z + (a + b)\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z + (a + b)\, f)'_{\restriction Z}(x) = \frac{x+a}{x-b}$.

(10) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z - (a + b)\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + b$ and $f_1(x) > 0$. Then $\operatorname{id}_Z - (a + b)\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z - (a + b)\, f)'_{\restriction Z}(x) = \frac{x-a}{x+b}$.

(11) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z + (b - a)\, f)$ and $f = \log_{_}(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - b$ and $f_1(x) > 0$. Then $\operatorname{id}_Z + (b - a)\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{id}_Z + (b - a)\, f)'_{\restriction Z}(x) = \frac{x-a}{x-b}$.

(12) Suppose $Z \subseteq \operatorname{dom}(f_1 + c\, f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$ and $f_2 = \frac{2}{\mathbb{Z}}$. Then $f_1 + c\, f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + c\, f_2)'_{\restriction Z}(x) = b + 2 \cdot c \cdot x$.

(13) Suppose $Z \subseteq \operatorname{dom}(\log_{_}(e) \cdot (f_1 + c\, f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$ and $(f_1 + c\, f_2)(x) > 0$. Then $\log_{_}(e) \cdot (f_1 + c\, f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot (f_1 + c\, f_2))'_{\restriction Z}(x) = \frac{b+2 \cdot c \cdot x}{a+b \cdot x+c \cdot x^2}$.

(14) Suppose $Z \subseteq \operatorname{dom} f$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) \neq 0$. Then $\frac{1}{f}$ is differentiable on Z and for every x such that

$x \in Z$ holds $(\frac{1}{f})'_{\restriction Z}(x) = -\frac{1}{(a+x)^{\mathbf{2}}}$.

(15) Suppose $Z \subseteq \operatorname{dom}((-1)\,\frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) \neq 0$. Then $(-1)\,\frac{1}{f}$ is differentiable on Z and for every x such that $x \in Z$ holds $((-1)\,\frac{1}{f})'_{\restriction Z}(x) = \frac{1}{(a+x)^{\mathbf{2}}}$.

(16) Suppose $Z \subseteq \operatorname{dom} f$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) \neq 0$. Then $\frac{1}{f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{f})'_{\restriction Z}(x) = \frac{1}{(a-x)^{\mathbf{2}}}$.

(17) Suppose $Z \subseteq \operatorname{dom}(f_1 + f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f_2 = \frac{2}{\mathbb{Z}}$. Then $f_1 + f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + f_2)'_{\restriction Z}(x) = 2 \cdot x$.

(18) Suppose $Z \subseteq \operatorname{dom}(\log_{_}(e) \cdot (f_1 + f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 + f_2)(x) > 0$. Then $\log_{_}(e) \cdot (f_1 + f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot (f_1 + f_2))'_{\restriction Z}(x) = \frac{2 \cdot x}{a^{\mathbf{2}} + x^2}$.

(19) Suppose $Z \subseteq \operatorname{dom}(-\log_{_}(e) \cdot (f_1 - f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 - f_2)(x) > 0$. Then $-\log_{_}(e) \cdot (f_1 - f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log_{_}(e) \cdot (f_1 - f_2))'_{\restriction Z}(x) = \frac{2 \cdot x}{a^{\mathbf{2}} - x^2}$.

(20) Suppose $Z \subseteq \operatorname{dom}(f_1 + f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a$ and $f_2 = \frac{3}{\mathbb{Z}}$. Then $f_1 + f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + f_2)'_{\restriction Z}(x) = 3 \cdot x^2$.

(21) Suppose $Z \subseteq \operatorname{dom}(\log_{_}(e) \cdot (f_1 + f_2))$ and $f_2 = \frac{3}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a$ and $(f_1 + f_2)(x) > 0$. Then $\log_{_}(e) \cdot (f_1 + f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot (f_1 + f_2))'_{\restriction Z}(x) = \frac{3 \cdot x^2}{a + x^3}$.

(22) Suppose $Z \subseteq \operatorname{dom}(\log_{_}(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$ and $f_2(x) = a - x$ and $f_2(x) > 0$. Then $\log_{_}(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2 \cdot a}{a^{\mathbf{2}} - x^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq \operatorname{dom}(\log_{_}(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x + a$ and $f_2(x) > 0$. Then $\log_{_}(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2 \cdot a}{x^{\mathbf{2}} - a^{\mathbf{2}}}$.

(24) Suppose $Z \subseteq \operatorname{dom}(\log_{_}(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x - b$ and $f_2(x) > 0$. Then $\log_{_}(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{a - b}{(x-a) \cdot (x-b)}$.

(25) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{a-b}\, f)$ and $f = \log_{_}(e) \cdot \frac{f_1}{f_2}$ and for every x such that

$x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x - b$ and $f_2(x) > 0$ and $a - b \neq 0$. Then $\frac{1}{a-b}\, f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{a-b}\, f)'_{\restriction Z}(x) = \frac{1}{(x-a)\cdot(x-b)}$.

(26) Suppose $Z \subseteq \operatorname{dom}(\log_-(e) \cdot \frac{f_1}{f_2})$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) > 0$ and $x \neq 0$. Then $\log_-(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_-(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2\cdot a - x}{x\cdot(x-a)}$.

(27) Suppose $Z \subseteq \operatorname{dom}((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $(\frac{3}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{3}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = \frac{3}{2} \cdot (a + x)^{\frac{1}{2}}_{\mathbb{R}}$.

(28) Suppose $Z \subseteq \operatorname{dom}(\frac{2}{3}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $\frac{2}{3}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + x)^{\frac{1}{2}}_{\mathbb{R}}$.

(29) Suppose $Z \subseteq \operatorname{dom}((-\frac{2}{3})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $(-\frac{2}{3})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-\frac{2}{3})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - x)^{\frac{1}{2}}_{\mathbb{R}}$.

(30) Suppose $Z \subseteq \operatorname{dom}(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + x)^{-\frac{1}{2}}_{\mathbb{R}}$.

(31) Suppose $Z \subseteq \operatorname{dom}((-2)\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $(-2)\,((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-2)\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - x)^{-\frac{1}{2}}_{\mathbb{R}}$.

(32) Suppose $Z \subseteq \operatorname{dom}(\frac{2}{3\cdot b}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + b \cdot x$ and $b \neq 0$ and $f(x) > 0$. Then $\frac{2}{3\cdot b}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3\cdot b}\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + b \cdot x)^{\frac{1}{2}}_{\mathbb{R}}$.

(33) Suppose $Z \subseteq \operatorname{dom}((-\frac{2}{3\cdot b})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - b \cdot x$ and $b \neq 0$ and $f(x) > 0$. Then $(-\frac{2}{3\cdot b})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-\frac{2}{3\cdot b})\,((\frac{3}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - b \cdot x)^{\frac{1}{2}}_{\mathbb{R}}$.

(34) Suppose $Z \subseteq \operatorname{dom}((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ and $f = f_1 + f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $(\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on

Z and for every x such that $x \in Z$ holds $((\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f)'_{\restriction Z}(x) = x \cdot (a^{\mathbf{2}} + x^2)^{-\frac{1}{2}}_{\mathbb{R}}$.

(35) Suppose $Z \subseteq \mathrm{dom}(-(\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $-(\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-(\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f)'_{\restriction Z}(x) = x \cdot (a^{\mathbf{2}} - x^2)^{-\frac{1}{2}}_{\mathbb{R}}$.

(36) Suppose $Z \subseteq \mathrm{dom}(2\,((\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f))$ and $f = f_1 + f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = x$ and $f(x) > 0$. Then $2\,((\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(2\,((\frac{1}{\mathbb{R}}^{\frac{2}{}}) \cdot f))'_{\restriction Z}(x) = (2 \cdot x + 1) \cdot (x^2 + x)^{-\frac{1}{2}}_{\mathbb{R}}$.

(37) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) (the function sin) $\cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot f)'_{\restriction Z}(x) = a \cdot (\text{the function cos})(a \cdot x + b)$.

(38) Suppose $Z \subseteq \mathrm{dom}((\text{the function cos}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) (the function cos) $\cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cos}) \cdot f)'_{\restriction Z}(x) = -a \cdot (\text{the function sin})(a \cdot x + b)$.

(39) Suppose that for every x such that $x \in Z$ holds $(\text{the function cos})(x) \neq 0$. Then

(i) $\frac{1}{\text{the function cos}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function cos}})'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(40) Suppose that for every x such that $x \in Z$ holds $(\text{the function sin})(x) \neq 0$. Then

(i) $\frac{1}{\text{the function sin}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function sin}})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(41) Suppose $Z \subseteq \mathrm{dom}((\text{the function sin})\ (\text{the function cos}))$. Then

(i) (the function sin) (the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function sin})\ (\text{the function cos}))'_{\restriction Z}(x) = \cos(2 \cdot x)$.

(42) Suppose $Z \subseteq \mathrm{dom}(\log_{_}(e) \cdot (\text{the function cos}))$ and for every x such that $x \in Z$ holds $(\text{the function cos})(x) > 0$. Then $\log_{_}(e) \cdot (\text{the function cos})$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{_}(e) \cdot (\text{the function cos}))'_{\restriction Z}(x) = -\tan x$.

(43) Suppose $Z \subseteq \operatorname{dom}(\log_{-}(e) \cdot (\text{the function sin}))$ and for every x such that $x \in Z$ holds $(\text{the function sin})(x) > 0$. Then $\log_{-}(e) \cdot (\text{the function sin})$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{-}(e) \cdot (\text{the function sin}))'_{\restriction Z}(x) = \cot x$.

(44) Suppose $Z \subseteq \operatorname{dom}((-\mathrm{id}_Z)\,(\text{the function cos}))$. Then

(i) $(-\mathrm{id}_Z)\,(\text{the function cos})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\mathrm{id}_Z)\,(\text{the function cos}))'_{\restriction Z}(x) = -(\text{the function cos})(x) + x \cdot (\text{the function sin})(x)$.

(45) Suppose $Z \subseteq \operatorname{dom}(\mathrm{id}_Z\,(\text{the function sin}))$. Then

(i) $\mathrm{id}_Z\,(\text{the function sin})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function sin}))'_{\restriction Z}(x) = (\text{the function sin})(x) + x \cdot (\text{the function cos})(x)$.

(46) Suppose $Z \subseteq \operatorname{dom}((-\mathrm{id}_Z)\,(\text{the function cos}) + \text{the function sin})$. Then

(i) $(-\mathrm{id}_Z)\,(\text{the function cos}) + \text{the function sin}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\mathrm{id}_Z)\,(\text{the function cos}) + \text{the function sin})'_{\restriction Z}(x) = x \cdot (\text{the function sin})(x)$.

(47) Suppose $Z \subseteq \operatorname{dom}(\mathrm{id}_Z\,(\text{the function sin}) + \text{the function cos})$. Then

(i) $\mathrm{id}_Z\,(\text{the function sin}) + \text{the function cos}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function sin}) + \text{the function cos})'_{\restriction Z}(x) = x \cdot (\text{the function cos})(x)$.

(48) Suppose $Z \subseteq \operatorname{dom}(2\,((\mathbb{R}^{\frac{1}{2}}) \cdot (\text{the function sin})))$ and for every x such that $x \in Z$ holds $(\text{the function sin})(x) > 0$. Then

(i) $2\,((\mathbb{R}^{\frac{1}{2}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(2\,((\mathbb{R}^{\frac{1}{2}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = (\text{the function cos})(x) \cdot (\text{the function sin})(x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(49) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin})))$. Then

(i) $\frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin}))$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin})))'_{\restriction Z}(x) = (\text{the function sin})(x) \cdot (\text{the function cos})(x)$.

(50) Suppose that

(i) $Z \subseteq \operatorname{dom}((\text{the function sin}) + \frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin})))$, and

(ii) for every x such that $x \in Z$ holds $(\text{the function sin})(x) > 0$ and $(\text{the function sin})(x) < 1$.

Then

(iii) $(\text{the function sin}) + \frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin}))$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $((\text{the function sin}) + \frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin})))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)^3}{1 - (\text{the function sin})(x)}$.

(51) Suppose that

(i) $Z \subseteq \operatorname{dom}(\frac{1}{2}\,((\mathbb{Z}^2) \cdot (\text{the function sin})) - \text{the function cos})$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function cos)$(x) < 1$.

Then

(iii) $\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot$ (the function sin))$-$the function cos is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot$ (the function sin))$-$the function cos$)'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)^3}{1-(\text{the function cos})(x)}$.

(52) Suppose that

(i) $Z \subseteq \operatorname{dom}((\text{the function sin})-\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function sin)$(x) > -1$.

Then

(iii) (the function sin)$-\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot$ (the function sin)) is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $((\text{the function sin})-\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)^3}{1+(\text{the function sin})(x)}$.

(53) Suppose that

(i) $Z \subseteq \operatorname{dom}(-\text{the function cos} - \frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function cos)$(x) > -1$.

Then

(iii) $-$the function cos $- \frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot$ (the function sin)) is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $(-\text{the function cos} - \frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)^3}{1+(\text{the function cos})(x)}$.

(54) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{n}\,((\frac{n}{\mathbb{Z}}) \cdot (\text{the function sin})))$ and $n > 0$. Then

(i) $\frac{1}{n}\,((\frac{n}{\mathbb{Z}}) \cdot$ (the function sin)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{n}\,((\frac{n}{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = ((\text{the function sin})(x)^{n-1}_{\mathbb{Z}}) \cdot (\text{the function cos})(x)$.

(55) Suppose $Z \subseteq \operatorname{dom}(\exp f)$ and for every x such that $x \in Z$ holds $f(x) = x - 1$. Then $\exp f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\exp f)'_{\restriction Z}(x) = x \cdot \exp(x)$.

(56) Suppose $Z \subseteq \operatorname{dom}(\log_{-}(e) \cdot \frac{\exp}{\exp + f})$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $\log_{-}(e) \cdot \frac{\exp}{\exp + f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{-}(e) \cdot \frac{\exp}{\exp + f})'_{\restriction Z}(x) = \frac{1}{\exp(x)+1}$.

(57) Suppose $Z \subseteq \operatorname{dom}(\log_{-}(e) \cdot \frac{\exp - f}{\exp})$ and for every x such that $x \in Z$ holds $f(x) = 1$ and $(\exp - f)(x) > 0$. Then $\log_{-}(e) \cdot \frac{\exp - f}{\exp}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_{-}(e) \cdot \frac{\exp - f}{\exp})'_{\restriction Z}(x) = \frac{1}{\exp(x)-1}$.

References

[1] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.
[2] Pacharapokin Chanapat, Kanchun, and Hiroshi Yamazaki. Formulas and identities of trigonometric functions. *Formalized Mathematics*, 12(**2**):139–141, 2004.
[3] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[4] Jarosław Kotowicz. Partial functions from a domain to a domain. *Formalized Mathematics*, 1(**4**):697–702, 1990.
[5] Jarosław Kotowicz. Partial functions from a domain to the set of real numbers. *Formalized Mathematics*, 1(**4**):703–709, 1990.
[6] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(**2**):269–272, 1990.
[7] Konrad Raczkowski. Integer and rational exponents. *Formalized Mathematics*, 2(**1**):125–130, 1991.
[8] Konrad Raczkowski and Andrzej Nędzusiak. Real exponents and logarithms. *Formalized Mathematics*, 2(**2**):213–216, 1991.
[9] Konrad Raczkowski and Paweł Sadowski. Real function differentiability. *Formalized Mathematics*, 1(**4**):797–801, 1990.
[10] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.
[11] Yasunari Shidama. The Taylor expansions. *Formalized Mathematics*, 12(**2**):195–200, 2004.
[12] Andrzej Trybulec. Subsets of complex numbers. *To appear in Formalized Mathematics.*
[13] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[14] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[15] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[16] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[17] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(**2**):255–263, 1998.

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