FORMALIZED MATHEMATICS
Volume 13, Number 1, 2005
University of Białystok

Propositional Calculus for Boolean Valued Functions. Part VIII

Shunichi Kobayashi
Matsumoto University
Nagano

Summary. In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

MML Identifier: BVFUNC26.

The articles [5], [6], [8], [7], [9], [1], [4], [3], and [2] provide the notation and terminology for this paper.

In this paper Y denotes a non empty set and a, b, c denote elements of $Boolean^Y$.

Let p, q be boolean-valued functions. The functor $p\ 'nand'\ q$ yielding a function is defined as follows:

(Def. 1) $\mathrm{dom}(p\ 'nand'\ q) = \mathrm{dom}\, p \cap \mathrm{dom}\, q$ and for every set x such that $x \in \mathrm{dom}(p\ 'nand'\ q)$ holds $(p\ 'nand'\ q)(x) = p(x)\ 'nand'\ q(x)$.

Let us observe that the functor $p\ 'nand'\ q$ is commutative. The functor $p\ 'nor'\ q$ yielding a function is defined as follows:

(Def. 2) $\mathrm{dom}(p\ 'nor'\ q) = \mathrm{dom}\, p \cap \mathrm{dom}\, q$ and for every set x such that $x \in \mathrm{dom}(p\ 'nor'\ q)$ holds $(p\ 'nor'\ q)(x) = p(x)\ 'nor'\ q(x)$.

Let us note that the functor $p\ 'nor'\ q$ is commutative.

Let p, q be boolean-valued functions. Note that $p\ 'nand'\ q$ is boolean-valued and $p\ 'nor'\ q$ is boolean-valued.

Let A be a non empty set and let p, q be elements of $Boolean^A$. Then $p\ 'nand'\ q$ is an element of $Boolean^A$ and it can be characterized by the condition:

(Def. 3) For every element x of A holds $(p\ 'nand'\ q)(x) = p(x)\ 'nand'\ q(x)$.

© 2005 University of Białystok
ISSN 1426–2630

Then $p\ 'nor'\ q$ is an element of $Boolean^A$ and it can be characterized by the condition:

(Def. 4) For every element x of A holds $(p\ 'nor'\ q)(x) = p(x)\ 'nor'\ q(x)$.

Let us consider Y and let a, b be elements of $\mathrm{BVF}(Y)$. Then $a\ 'nand'\ b$ is an element of $\mathrm{BVF}(Y)$. Then $a\ 'nor'\ b$ is an element of $\mathrm{BVF}(Y)$.

We now state a number of propositions:

(1) $a\ 'nand'\ b = \neg(a \wedge b)$.

(2) $a\ 'nor'\ b = \neg(a \vee b)$.

(3) $true(Y)\ 'nand'\ a = \neg a$.

(4) $false(Y)\ 'nand'\ a = true(Y)$.

(5) $false(Y)\ 'nand'\ false(Y) = true(Y)$ and $false(Y)\ 'nand'\ true(Y) = true(Y)$ and $true(Y)\ 'nand'\ true(Y) = false(Y)$.

(6) $a\ 'nand'\ a = \neg a$ and $\neg(a\ 'nand'\ a) = a$.

(7) $\neg(a\ 'nand'\ b) = a \wedge b$.

(8) $a\ 'nand'\ \neg a = true(Y)$ and $\neg(a\ 'nand'\ \neg a) = false(Y)$.

(9) $a\ 'nand'\ b \wedge c = \neg(a \wedge b \wedge c)$.

(10) $a\ 'nand'\ b \wedge c = a \wedge b\ 'nand'\ c$.

(11) $a\ 'nand'\ (b \vee c) = \neg(a \wedge b) \wedge \neg(a \wedge c)$.

(12) $a\ 'nand'\ (b \oplus c) = a \wedge b \Leftrightarrow a \wedge c$.

(13) $a\ 'nand'\ (b\ 'nand'\ c) = \neg a \vee b \wedge c$ and $a\ 'nand'\ (b\ 'nand'\ c) = a \Rightarrow b \wedge c$.

(14) $a\ 'nand'\ (b\ 'nor'\ c) = \neg a \vee b \vee c$ and $a\ 'nand'\ (b\ 'nor'\ c) = a \Rightarrow b \vee c$.

(15) $a\ 'nand'\ (b \Leftrightarrow c) = a \Rightarrow b \oplus c$.

(16) $a\ 'nand'\ a \wedge b = a\ 'nand'\ b$.

(17) $a\ 'nand'\ (a \vee b) = \neg a \wedge \neg(a \wedge b)$.

(18) $a\ 'nand'\ (a \Leftrightarrow b) = a \Rightarrow a \oplus b$.

(19) $a\ 'nand'\ (a\ 'nand'\ b) = \neg a \vee b$ and $a\ 'nand'\ (a\ 'nand'\ b) = a \Rightarrow b$.

(20) $a\ 'nand'\ (a\ 'nor'\ b) = true(Y)$.

(21) $a\ 'nand'\ (a \Leftrightarrow b) = \neg a \vee \neg b$.

(22) $a \wedge b = a\ 'nand'\ b\ 'nand'\ (a\ 'nand'\ b)$.

(23) $a\ 'nand'\ b\ 'nand'\ (a\ 'nand'\ c) = a \wedge (b \vee c)$.

(24) $a\ 'nand'\ (b \Rightarrow c) = (\neg a \vee b) \wedge \neg(a \wedge c)$.

(25) $a\ 'nand'\ (a \Rightarrow b) = \neg(a \wedge b)$.

(26) $true(Y)\ 'nor'\ a = false(Y)$.

(27) $false(Y)\ 'nor'\ a = \neg a$.

(28) $false(Y)\ 'nor'\ false(Y) = true(Y)$ and $false(Y)\ 'nor'\ true(Y) = false(Y)$ and $true(Y)\ 'nor'\ true(Y) = false(Y)$.

(29) $a\ 'nor'\ a = \neg a$ and $\neg(a\ 'nor'\ a) = a$.

(30) $\neg(a\ {}'\mathrm{nor}'\ b) = a \vee b.$

(31) $a\ {}'\mathrm{nor}'\ \neg a = \mathit{false}(Y)$ and $\neg(a\ {}'\mathrm{nor}'\ \neg a) = \mathit{true}(Y).$

(32) $\neg a \wedge (a \oplus b) = \neg a \wedge b.$

(33) $a\ {}'\mathrm{nor}'\ b \wedge c = \neg(a \vee b) \vee \neg(a \vee c).$

(34) $a\ {}'\mathrm{nor}'\ (b \vee c) = \neg(a \vee b \vee c).$

(35) $a\ {}'\mathrm{nor}'\ (b \Leftrightarrow c) = \neg a \wedge (b \oplus c).$

(36) $a\ {}'\mathrm{nor}'\ (b \Rightarrow c) = \neg a \wedge b \wedge \neg c.$

(37) $a\ {}'\mathrm{nor}'\ (b\ {}'\mathrm{nand}'\ c) = \neg a \wedge b \wedge c.$

(38) $a\ {}'\mathrm{nor}'\ (b\ {}'\mathrm{nor}'\ c) = \neg a \wedge (b \vee c).$

(39) $a\ {}'\mathrm{nor}'\ a \wedge b = \neg(a \wedge (a \vee b)).$

(40) $a\ {}'\mathrm{nor}'\ (a \vee b) = \neg(a \vee b).$

(41) $a\ {}'\mathrm{nor}'\ (a \Leftrightarrow b) = \neg a \wedge b.$

(42) $a\ {}'\mathrm{nor}'\ (a \Rightarrow b) = \mathit{false}(Y).$

(43) $a\ {}'\mathrm{nor}'\ (a\ {}'\mathrm{nand}'\ b) = \mathit{false}(Y).$

(44) $a\ {}'\mathrm{nor}'\ (a\ {}'\mathrm{nor}'\ b) = \neg a \wedge b.$

(45) $\mathit{false}(Y) \Leftrightarrow \mathit{false}(Y) = \mathit{true}(Y).$

(46) $\mathit{false}(Y) \Leftrightarrow \mathit{true}(Y) = \mathit{false}(Y).$

(47) $\mathit{true}(Y) \Leftrightarrow \mathit{true}(Y) = \mathit{true}(Y).$

(48) $a \Leftrightarrow a = \mathit{true}(Y)$ and $\neg(a \Leftrightarrow a) = \mathit{false}(Y).$

(49) $a \Leftrightarrow a \vee b = a \vee \neg b.$

(50) $a \wedge (b\ {}'\mathrm{nand}'\ c) = a \wedge \neg b \vee a \wedge \neg c.$

(51) $a \vee (b\ {}'\mathrm{nand}'\ c) = a \vee \neg b \vee \neg c.$

(52) $a \oplus (b\ {}'\mathrm{nand}'\ c) = \neg a \wedge \neg(b \wedge c) \vee a \wedge b \wedge c.$

(53) $a \Leftrightarrow b\ {}'\mathrm{nand}'\ c = a \wedge \neg(b \wedge c) \vee \neg a \wedge b \wedge c.$

(54) $a \Rightarrow b\ {}'\mathrm{nand}'\ c = \neg(a \wedge b \wedge c).$

(55) $a\ {}'\mathrm{nor}'\ (b\ {}'\mathrm{nand}'\ c) = \neg(a \vee \neg b \vee \neg c).$

(56) $a \wedge (a\ {}'\mathrm{nand}'\ b) = a \wedge \neg b.$

(57) $a \vee (a\ {}'\mathrm{nand}'\ b) = \mathit{true}(Y).$

(58) $a \oplus (a\ {}'\mathrm{nand}'\ b) = \neg a \vee b.$

(59) $a \Leftrightarrow a\ {}'\mathrm{nand}'\ b = a \wedge \neg b.$

(60) $a \Rightarrow a\ {}'\mathrm{nand}'\ b = \neg(a \wedge b).$

(61) $a\ {}'\mathrm{nor}'\ (a\ {}'\mathrm{nand}'\ b) = \mathit{false}(Y).$

(62) $a \wedge (b\ {}'\mathrm{nor}'\ c) = a \wedge \neg b \wedge \neg c.$

(63) $a \vee (b\ {}'\mathrm{nor}'\ c) = (a \vee \neg b) \wedge (a \vee \neg c).$

(64) $a \oplus (b\ {}'\mathrm{nor}'\ c) = (a \vee \neg(b \vee c)) \wedge (\neg a \vee b \vee c).$

(65) $a \Leftrightarrow b\ {}'\mathrm{nor}'\ c = (a \vee b \vee c) \wedge (\neg a \vee \neg(b \vee c)).$

(66) $a \Rightarrow b\ {}'\mathrm{nor}'\ c = \neg(a \wedge (b \vee c)).$

(67) $a \ '\mathrm{nand}'\ (b \ '\mathrm{nor}'\ c) = \neg a \vee b \vee c$.

(68) $a \wedge (a \ '\mathrm{nor}'\ b) = \mathit{false}(Y)$.

(69) $a \vee (a \ '\mathrm{nor}'\ b) = a \vee \neg b$.

(70) $a \oplus (a \ '\mathrm{nor}'\ b) = a \vee \neg b$.

(71) $a \Leftrightarrow a \ '\mathrm{nor}'\ b = \neg a \wedge b$.

(72) $a \Rightarrow a \ '\mathrm{nor}'\ b = \neg(a \vee a \wedge b)$.

(73) $a \ '\mathrm{nand}'\ (a \ '\mathrm{nor}'\ b) = \mathit{true}(Y)$.

Acknowledgments

This research was partially supported by the research funds of the Matsumoto University.

References

[1] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[2] Shunichi Kobayashi. On the calculus of binary arithmetics. *Formalized Mathematics*, 11(**4**):417–419, 2003.
[3] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(**2**):249–254, 1998.
[4] Andrzej Trybulec. Function domains and Frænkel operator. *Formalized Mathematics*, 1(**3**):495–500, 1990.
[5] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[6] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[7] Edmund Woronowicz. Interpretation and satisfiability in the first order logic. *Formalized Mathematics*, 1(**4**):739–743, 1990.
[8] Edmund Woronowicz. Many–argument relations. *Formalized Mathematics*, 1(**4**):733–737, 1990.
[9] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received September 5, 2004