

Several Differentiation Formulas of Special Functions. Part III

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Summary. In this article, we give several differentiation formulas of special and composite functions including trigonometric function, inverse trigonometric function, polynomial function and logarithmic function.

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The articles [13], [15], [16], [1], [4], [10], [11], [17], [5], [14], [12], [2], [6], [9], [7], [8], and [3] provide the terminology and notation for this paper.

For simplicity, we follow the rules: x, r, a, b denote real numbers, n denotes a natural number, Z denotes an open subset of $\mathbb{R}$, and f, f_1, f_2, f_3 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

- (1) $x_{\mathbb{Z}}^2 = x^2$.
- (2) If $x > 0$, then $x_{\mathbb{R}}^{\frac{1}{2}} = \sqrt{x}$.
- (3) If $x > 0$, then $x_{\mathbb{R}}^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$.
- (4) Suppose $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}(r \text{ (the function arcsin)})$. Then
 - (i) $r \text{ (the function arcsin)}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(r \text{ (the function arcsin)})'_{|Z}(x) = \frac{r}{\sqrt{1-x^2}}$.

- (5) Suppose $Z \subseteq]-1, 1[$ and $Z \subseteq \text{dom}(r(\text{the function arccos}))$. Then
- (i) $r(\text{the function arccos})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(r(\text{the function arccos}))'_{|Z}(x) = -\frac{r}{\sqrt{1-x^2}}$.
- (6) Suppose f is differentiable in x and $f(x) > -1$ and $f(x) < 1$. Then $(\text{the function arcsin}) \cdot f$ is differentiable in x and $((\text{the function arcsin}) \cdot f)'(x) = \frac{f'(x)}{\sqrt{1-f(x)^2}}$.
- (7) Suppose f is differentiable in x and $f(x) > -1$ and $f(x) < 1$. Then $(\text{the function arccos}) \cdot f$ is differentiable in x and $((\text{the function arccos}) \cdot f)'(x) = -\frac{f'(x)}{\sqrt{1-f(x)^2}}$.
- (8) Suppose $Z \subseteq \text{dom}(\log_-(e) \cdot (\text{the function arcsin}))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $(\text{the function arcsin})(x) > 0$. Then
- (i) $\log_-(e) \cdot (\text{the function arcsin})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\log_-(e) \cdot (\text{the function arcsin}))'_{|Z}(x) = \frac{1}{\sqrt{1-x^2} \cdot (\text{the function arcsin})(x)}$.
- (9) Suppose $Z \subseteq \text{dom}(\log_-(e) \cdot (\text{the function arccos}))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $(\text{the function arccos})(x) > 0$. Then
- (i) $\log_-(e) \cdot (\text{the function arccos})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\log_-(e) \cdot (\text{the function arccos}))'_{|Z}(x) = -\frac{1}{\sqrt{1-x^2} \cdot (\text{the function arccos})(x)}$.
- (10) Suppose $Z \subseteq \text{dom}(\binom{n}{\mathbb{Z}} \cdot (\text{the function arcsin}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $\binom{n}{\mathbb{Z}} \cdot (\text{the function arcsin})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\binom{n}{\mathbb{Z}} \cdot (\text{the function arcsin}))'_{|Z}(x) = \frac{n \cdot (\text{the function arcsin})(x)_{\mathbb{Z}}^{n-1}}{\sqrt{1-x^2}}$.
- (11) Suppose $Z \subseteq \text{dom}(\binom{n}{\mathbb{Z}} \cdot (\text{the function arccos}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $\binom{n}{\mathbb{Z}} \cdot (\text{the function arccos})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\binom{n}{\mathbb{Z}} \cdot (\text{the function arccos}))'_{|Z}(x) = -\frac{n \cdot (\text{the function arccos})(x)_{\mathbb{Z}}^{n-1}}{\sqrt{1-x^2}}$.
- (12) Suppose $Z \subseteq \text{dom}(\frac{1}{2}(\binom{2}{\mathbb{Z}} \cdot (\text{the function arcsin})))$ and $Z \subseteq]-1, 1[$. Then
- (i) $\frac{1}{2}(\binom{2}{\mathbb{Z}} \cdot (\text{the function arcsin}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{2}(\binom{2}{\mathbb{Z}} \cdot (\text{the function arcsin})))'_{|Z}(x) = \frac{(\text{the function arcsin})(x)}{\sqrt{1-x^2}}$.
- (13) Suppose $Z \subseteq \text{dom}(\frac{1}{2}(\binom{2}{\mathbb{Z}} \cdot (\text{the function arccos})))$ and $Z \subseteq]-1, 1[$. Then
- (i) $\frac{1}{2}(\binom{2}{\mathbb{Z}} \cdot (\text{the function arccos}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{2}(\binom{2}{\mathbb{Z}} \cdot (\text{the function arccos})))'_{|Z}(x) = -\frac{(\text{the function arccos})(x)}{\sqrt{1-x^2}}$.

- (14) Suppose $Z \subseteq \text{dom}((\text{the function } \arcsin) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$ and $f(x) > -1$ and $f(x) < 1$. Then
- (i) $(\text{the function } \arcsin) \cdot f$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \arcsin) \cdot f)'_{|Z}(x) = \frac{a}{\sqrt{1-(a \cdot x + b)^2}}$.
- (15) Suppose $Z \subseteq \text{dom}((\text{the function } \arccos) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$ and $f(x) > -1$ and $f(x) < 1$. Then
- (i) $(\text{the function } \arccos) \cdot f$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \arccos) \cdot f)'_{|Z}(x) = -\frac{a}{\sqrt{1-(a \cdot x + b)^2}}$.
- (16) Suppose $Z \subseteq \text{dom}(\text{id}_Z (\text{the function } \arcsin))$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{id}_Z (\text{the function } \arcsin)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{id}_Z (\text{the function } \arcsin))'_{|Z}(x) = (\text{the function } \arcsin)(x) + \frac{x}{\sqrt{1-x^2}}$.
- (17) Suppose $Z \subseteq \text{dom}(\text{id}_Z (\text{the function } \arccos))$ and $Z \subseteq]-1, 1[$. Then
- (i) $\text{id}_Z (\text{the function } \arccos)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\text{id}_Z (\text{the function } \arccos))'_{|Z}(x) = (\text{the function } \arccos)(x) - \frac{x}{\sqrt{1-x^2}}$.
- (18) Suppose $Z \subseteq \text{dom}(f (\text{the function } \arcsin))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
- (i) $f (\text{the function } \arcsin)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(f (\text{the function } \arcsin))'_{|Z}(x) = a \cdot (\text{the function } \arcsin)(x) + \frac{a \cdot x + b}{\sqrt{1-x^2}}$.
- (19) Suppose $Z \subseteq \text{dom}(f (\text{the function } \arccos))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
- (i) $f (\text{the function } \arccos)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(f (\text{the function } \arccos))'_{|Z}(x) = a \cdot (\text{the function } \arccos)(x) - \frac{a \cdot x + b}{\sqrt{1-x^2}}$.
- (20) Suppose $Z \subseteq \text{dom}(\frac{1}{2} ((\text{the function } \arcsin) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = 2 \cdot x$ and $f(x) > -1$ and $f(x) < 1$. Then
- (i) $\frac{1}{2} ((\text{the function } \arcsin) \cdot f)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{2} ((\text{the function } \arcsin) \cdot f))'_{|Z}(x) = \frac{1}{\sqrt{1-(2 \cdot x)^2}}$.
- (21) Suppose $Z \subseteq \text{dom}(\frac{1}{2} ((\text{the function } \arccos) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = 2 \cdot x$ and $f(x) > -1$ and $f(x) < 1$. Then
- (i) $\frac{1}{2} ((\text{the function } \arccos) \cdot f)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{2} ((\text{the function } \arccos) \cdot f))'_{|Z}(x) = -\frac{1}{\sqrt{1-(2 \cdot x)^2}}$.

(22) Suppose $Z \subseteq \text{dom}((\frac{1}{\mathbb{R}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$. Then $(\frac{1}{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{1}{\mathbb{R}}) \cdot f)'_{|Z}(x) = -x \cdot (1 - x_{\mathbb{Z}}^2)_{\mathbb{R}}^{-\frac{1}{2}}$.

(23) Suppose that

- (i) $Z \subseteq \text{dom}(\text{id}_Z(\text{the function arcsin}) + (\frac{1}{\mathbb{R}}) \cdot f)$,
- (ii) $Z \subseteq]-1, 1[$,
- (iii) $f = f_1 - f_2$,
- (iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
- (v) for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$ and $x \neq 0$.

Then

- (vi) $\text{id}_Z(\text{the function arcsin}) + (\frac{1}{\mathbb{R}}) \cdot f$ is differentiable on Z , and
- (vii) for every x such that $x \in Z$ holds $(\text{id}_Z(\text{the function arcsin}) + (\frac{1}{\mathbb{R}}) \cdot f)'_{|Z}(x) = (\text{the function arcsin})(x)$.

(24) Suppose that

- (i) $Z \subseteq \text{dom}(\text{id}_Z(\text{the function arccos}) - (\frac{1}{\mathbb{R}}) \cdot f)$,
- (ii) $Z \subseteq]-1, 1[$,
- (iii) $f = f_1 - f_2$,
- (iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
- (v) for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f(x) > 0$ and $x \neq 0$.

Then

- (vi) $\text{id}_Z(\text{the function arccos}) - (\frac{1}{\mathbb{R}}) \cdot f$ is differentiable on Z , and
- (vii) for every x such that $x \in Z$ holds $(\text{id}_Z(\text{the function arccos}) - (\frac{1}{\mathbb{R}}) \cdot f)'_{|Z}(x) = (\text{the function arccos})(x)$.

(25) Suppose $Z \subseteq \text{dom}(\text{id}_Z((\text{the function arcsin}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = \frac{x}{a}$ and $f(x) > -1$ and $f(x) < 1$. Then

- (i) $\text{id}_Z((\text{the function arcsin}) \cdot f)$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\text{id}_Z((\text{the function arcsin}) \cdot f))'_{|Z}(x) = (\text{the function arcsin})(\frac{x}{a}) + \frac{x}{a \cdot \sqrt{1 - (\frac{x}{a})^2}}$.

(26) Suppose $Z \subseteq \text{dom}(\text{id}_Z((\text{the function arccos}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = \frac{x}{a}$ and $f(x) > -1$ and $f(x) < 1$. Then

- (i) $\text{id}_Z((\text{the function arccos}) \cdot f)$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\text{id}_Z((\text{the function arccos}) \cdot f))'_{|Z}(x) = (\text{the function arccos})(\frac{x}{a}) - \frac{x}{a \cdot \sqrt{1 - (\frac{x}{a})^2}}$.

(27) Suppose $Z \subseteq \text{dom}((\frac{1}{\mathbb{R}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^2$ and $f(x) > 0$. Then $(\frac{1}{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\frac{1}{\mathbb{R}}) \cdot f)'_{|Z}(x) = -x \cdot (a^2 - x_{\mathbb{Z}}^2)_{\mathbb{R}}^{-\frac{1}{2}}$.

(28) Suppose that

- (i) $Z \subseteq \text{dom}(\text{id}_Z ((\text{the function arcsin}) \cdot f_3) + (\frac{1}{\mathbb{R}}) \cdot f)$,
- (ii) $Z \subseteq]-1, 1[$,
- (iii) $f = f_1 - f_2$,
- (iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
- (v) for every x such that $x \in Z$ holds $f_1(x) = a^2$ and $f(x) > 0$ and $f_3(x) = \frac{x}{a}$ and $f_3(x) > -1$ and $f_3(x) < 1$ and $x \neq 0$ and $a > 0$.

Then

- (vi) $\text{id}_Z ((\text{the function arcsin}) \cdot f_3) + (\frac{1}{\mathbb{R}}) \cdot f$ is differentiable on Z , and
- (vii) for every x such that $x \in Z$ holds $(\text{id}_Z ((\text{the function arcsin}) \cdot f_3) + (\frac{1}{\mathbb{R}}) \cdot f)'_{|Z}(x) = (\text{the function arcsin})(\frac{x}{a})$.

(29) Suppose that

- (i) $Z \subseteq \text{dom}(\text{id}_Z ((\text{the function arccos}) \cdot f_3) - (\frac{1}{\mathbb{R}}) \cdot f)$,
- (ii) $Z \subseteq]-1, 1[$,
- (iii) $f = f_1 - f_2$,
- (iv) $f_2 = \frac{2}{\mathbb{Z}}$, and
- (v) for every x such that $x \in Z$ holds $f_1(x) = a^2$ and $f(x) > 0$ and $f_3(x) = \frac{x}{a}$ and $f_3(x) > -1$ and $f_3(x) < 1$ and $x \neq 0$ and $a > 0$.

Then

- (vi) $\text{id}_Z ((\text{the function arccos}) \cdot f_3) - (\frac{1}{\mathbb{R}}) \cdot f$ is differentiable on Z , and
- (vii) for every x such that $x \in Z$ holds $(\text{id}_Z ((\text{the function arccos}) \cdot f_3) - (\frac{1}{\mathbb{R}}) \cdot f)'_{|Z}(x) = (\text{the function arccos})(\frac{x}{a})$.

(30) Suppose $Z \subseteq \text{dom}((-\frac{1}{n})((\frac{n}{\mathbb{Z}}) \cdot \frac{1}{\text{the function sin}}))$ and $n > 0$ and for every x such that $x \in Z$ holds $(\text{the function sin})(x) \neq 0$. Then

- (i) $(-\frac{1}{n})((\frac{n}{\mathbb{Z}}) \cdot \frac{1}{\text{the function sin}})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((-\frac{1}{n})((\frac{n}{\mathbb{Z}}) \cdot \frac{1}{\text{the function sin}}))'_{|Z}(x) = \frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}}$.

(31) Suppose $Z \subseteq \text{dom}(\frac{1}{n}((\frac{n}{\mathbb{Z}}) \cdot \frac{1}{\text{the function cos}}))$ and $n > 0$ and for every x such that $x \in Z$ holds $(\text{the function cos})(x) \neq 0$. Then

- (i) $\frac{1}{n}((\frac{n}{\mathbb{Z}}) \cdot \frac{1}{\text{the function cos}})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\frac{1}{n}((\frac{n}{\mathbb{Z}}) \cdot \frac{1}{\text{the function cos}}))'_{|Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}}$.

(32) Suppose $Z \subseteq \text{dom}((\text{the function sin}) \cdot \log_-(e))$ and for every x such that $x \in Z$ holds $x > 0$. Then

- (i) $(\text{the function sin}) \cdot \log_-(e)$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot \log_-(e))'_{|Z}(x) = \frac{(\text{the function cos})(\log_-(e))(x)}{x}$.

(33) Suppose $Z \subseteq \text{dom}((\text{the function cos}) \cdot \log_-(e))$ and for every x such that $x \in Z$ holds $x > 0$. Then

- (i) (the function $\cos$) $\cdot \log_-(e)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot \log_-(e))'_{|Z}(x) = -\frac{(\text{the function } \sin)((\log_-(e))(x))}{x}$.
- (34) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) \cdot (\text{the function } \exp))$. Then
- (i) (the function $\sin$) $\cdot$ (the function $\exp$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot (\text{the function } \exp))'_{|Z}(x) = (\text{the function } \exp)(x) \cdot (\text{the function } \cos)((\text{the function } \exp)(x))$.
- (35) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) \cdot (\text{the function } \exp))$. Then
- (i) (the function $\cos$) $\cdot$ (the function $\exp$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot (\text{the function } \exp))'_{|Z}(x) = -(\text{the function } \exp)(x) \cdot (\text{the function } \sin)((\text{the function } \exp)(x))$.
- (36) Suppose $Z \subseteq \text{dom}((\text{the function } \exp) \cdot (\text{the function } \cos))$. Then
- (i) (the function $\exp$) $\cdot$ (the function $\cos$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) \cdot (\text{the function } \cos))'_{|Z}(x) = -(\text{the function } \exp)((\text{the function } \cos)(x)) \cdot (\text{the function } \sin)(x)$.
- (37) Suppose $Z \subseteq \text{dom}((\text{the function } \exp) \cdot (\text{the function } \sin))$. Then
- (i) (the function $\exp$) $\cdot$ (the function $\sin$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) \cdot (\text{the function } \sin))'_{|Z}(x) = (\text{the function } \exp)((\text{the function } \sin)(x)) \cdot (\text{the function } \cos)(x)$.
- (38) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) + (\text{the function } \cos))$. Then
- (i) (the function $\sin$) $+$ (the function $\cos$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) + (\text{the function } \cos))'_{|Z}(x) = (\text{the function } \cos)(x) - (\text{the function } \sin)(x)$.
- (39) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) - (\text{the function } \cos))$. Then
- (i) (the function $\sin$) $-$ (the function $\cos$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) - (\text{the function } \cos))'_{|Z}(x) = (\text{the function } \cos)(x) + (\text{the function } \sin)(x)$.
- (40) Suppose $Z \subseteq \text{dom}((\text{the function } \exp) ((\text{the function } \sin) - (\text{the function } \cos)))$. Then
- (i) (the function $\exp$) $((\text{the function } \sin) - (\text{the function } \cos))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) ((\text{the function } \sin) - (\text{the function } \cos)))'_{|Z}(x) = 2 \cdot (\text{the function } \exp)(x) \cdot (\text{the function } \sin)(x)$.
- (41) Suppose $Z \subseteq \text{dom}((\text{the function } \exp) ((\text{the function } \sin) + (\text{the function } \cos)))$. Then

- (i) (the function exp) ((the function sin)+(the function cos)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function exp) ((the function sin)+(the function cos)))' $\upharpoonright_Z(x) = 2 \cdot (\text{the function exp})(x) \cdot (\text{the function cos})(x)$.
- (42) Suppose $Z \subseteq \text{dom}(\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})$. Then
- (i) $\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})' \upharpoonright_Z(x) = -\frac{2 \cdot (\text{the function sin})(x)}{(\text{the function exp})(x)}$.
- (43) Suppose $Z \subseteq \text{dom}(\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})$. Then
- (i) $\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})' \upharpoonright_Z(x) = \frac{2 \cdot (\text{the function cos})(x)}{(\text{the function exp})(x)}$.
- (44) Suppose $Z \subseteq \text{dom}((\text{the function exp}) (\text{the function sin}))$. Then
- (i) (the function exp) (the function sin) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function exp) (the function sin))' $\upharpoonright_Z(x) = (\text{the function exp})(x) \cdot ((\text{the function sin})(x) + (\text{the function cos})(x))$.
- (45) Suppose $Z \subseteq \text{dom}((\text{the function exp}) (\text{the function cos}))$. Then
- (i) (the function exp) (the function cos) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function exp) (the function cos))' $\upharpoonright_Z(x) = (\text{the function exp})(x) \cdot ((\text{the function cos})(x) - (\text{the function sin})(x))$.
- (46) Suppose $(\text{the function cos})(x) \neq 0$. Then
- (i) $\frac{\text{the function sin}}{\text{the function cos}}$ is differentiable in x , and
 - (ii) $(\frac{\text{the function sin}}{\text{the function cos}})'(x) = \frac{1}{(\text{the function cos})(x)^2}$.
- (47) Suppose $(\text{the function sin})(x) \neq 0$. Then
- (i) $\frac{\text{the function cos}}{\text{the function sin}}$ is differentiable in x , and
 - (ii) $(\frac{\text{the function cos}}{\text{the function sin}})'(x) = -\frac{1}{(\text{the function sin})(x)^2}$.
- (48) Suppose $Z \subseteq \text{dom}((\frac{2}{Z}) \cdot \frac{\text{the function sin}}{\text{the function cos}})$ and for every x such that $x \in Z$ holds $(\text{the function cos})(x) \neq 0$. Then
- (i) $(\frac{2}{Z}) \cdot \frac{\text{the function sin}}{\text{the function cos}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\frac{2}{Z}) \cdot \frac{\text{the function sin}}{\text{the function cos}})' \upharpoonright_Z(x) = \frac{2 \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\frac{3}{Z}}}$.
- (49) Suppose $Z \subseteq \text{dom}((\frac{2}{Z}) \cdot \frac{\text{the function cos}}{\text{the function sin}})$ and for every x such that $x \in Z$ holds $(\text{the function sin})(x) \neq 0$. Then
- (i) $(\frac{2}{Z}) \cdot \frac{\text{the function cos}}{\text{the function sin}}$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $((\frac{2}{Z}) \cdot \frac{\text{the function cos}}{\text{the function sin}})'|_Z(x) = \frac{2 \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^2}$.
- (50) Suppose that
- (i) $Z \subseteq \text{dom}(\frac{\text{the function sin}}{\text{the function cos}} \cdot f)$, and
- (ii) for every x such that $x \in Z$ holds $f(x) = \frac{x}{2}$ and $(\text{the function cos})(f(x)) \neq 0$.
- Then
- (iii) $\frac{\text{the function sin}}{\text{the function cos}} \cdot f$ is differentiable on Z , and
- (iv) for every x such that $x \in Z$ holds $(\frac{\text{the function sin}}{\text{the function cos}} \cdot f)'|_Z(x) = \frac{1}{1 + (\text{the function cos})(x)}$.
- (51) Suppose that
- (i) $Z \subseteq \text{dom}(\frac{\text{the function cos}}{\text{the function sin}} \cdot f)$, and
- (ii) for every x such that $x \in Z$ holds $f(x) = \frac{x}{2}$ and $(\text{the function sin})(f(x)) \neq 0$.
- Then
- (iii) $\frac{\text{the function cos}}{\text{the function sin}} \cdot f$ is differentiable on Z , and
- (iv) for every x such that $x \in Z$ holds $(\frac{\text{the function cos}}{\text{the function sin}} \cdot f)'|_Z(x) = \frac{1}{1 - (\text{the function cos})(x)}$.

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