

Partial Sum of Some Series

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Summary. Solving the partial sum of some often used series.

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The articles [2], [1], [4], [3], [5], [7], and [6] provide the notation and terminology for this paper.

In this paper n is a natural number and s is a sequence of real numbers.

Next we state a number of propositions:

- (1) $|(-1)^n| = 1$.
- (2) $(n+1)^3 = n^3 + 3 \cdot n^2 + 3 \cdot n + 1$ and $(n+1)^4 = n^4 + 4 \cdot n^3 + 6 \cdot n^2 + 4 \cdot n + 1$
and $(n+1)^5 = n^5 + 5 \cdot n^4 + 10 \cdot n^3 + 10 \cdot n^2 + 5 \cdot n + 1$.
- (3) If for every n holds $s(n) = n$, then for every n holds
 $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1)}{2}$.
- (4) If for every n holds $s(n) = 2 \cdot n$, then for every n holds
 $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = n \cdot (n+1)$.
- (5) If for every n holds $s(n) = 2 \cdot n + 1$, then for every n holds
 $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = (n+1)^2$.
- (6) If for every n holds $s(n) = n \cdot (n+1)$, then for every n holds
 $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (n+2)}{3}$.
- (7) If for every n holds $s(n) = n \cdot (n+1) \cdot (n+2)$, then for every n holds
 $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (n+2) \cdot (n+3)}{4}$.
- (8) If for every n holds $s(n) = n \cdot (n+1) \cdot (n+2) \cdot (n+3)$, then for every n
holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (n+2) \cdot (n+3) \cdot (n+4)}{5}$.
- (9) If for every n holds $s(n) = \frac{1}{n \cdot (n+1)}$, then for every n holds
 $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{n+1}$.

- (10) If for every n holds $s(n) = \frac{1}{n \cdot (n+1) \cdot (n+2)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{4} - \frac{1}{2 \cdot (n+1) \cdot (n+2)}$.
- (11) If for every n holds $s(n) = \frac{1}{n \cdot (n+1) \cdot (n+2) \cdot (n+3)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{18} - \frac{1}{3 \cdot (n+1) \cdot (n+2) \cdot (n+3)}$.
- (12) If for every n holds $s(n) = n^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (2 \cdot n+1)}{6}$.
- (13) If for every n holds $s(n) = (-1)^{n+1} \cdot n^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{(-1)^{n+1} \cdot n \cdot (n+1)}{2}$.
- (14) If for every n such that $n \geq 1$ holds $s(n) = (2 \cdot n - 1)^2$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (4 \cdot n^2 - 1)}{3}$.
- (15) If for every n holds $s(n) = n^3$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n^2 \cdot (n+1)^2}{4}$.
- (16) If for every n such that $n \geq 1$ holds $s(n) = (2 \cdot n - 1)^3$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = n^2 \cdot (2 \cdot n^2 - 1)$.
- (17) If for every n holds $s(n) = n^4$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (2 \cdot n+1) \cdot ((3 \cdot n^2 + 3 \cdot n) - 1)}{30}$.
- (18) If for every n holds $s(n) = (-1)^{n+1} \cdot n^4$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{(-1)^{n+1} \cdot n \cdot (n+1) \cdot ((n^2 + n) - 1)}{2}$.
- (19) If for every n holds $s(n) = n^5$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n^2 \cdot (n+1)^2 \cdot ((2 \cdot n^2 + 2 \cdot n) - 1)}{12}$.
- (20) If for every n holds $s(n) = n^6$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (2 \cdot n+1) \cdot (((3 \cdot n^4 + 6 \cdot n^3) - 3 \cdot n) + 1)}{42}$.
- (21) If for every n holds $s(n) = n^7$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n^2 \cdot (n+1)^2 \cdot (((3 \cdot n^4 + 6 \cdot n^3) - n^2 - 4 \cdot n) + 2)}{24}$.
- (22) If for every n holds $s(n) = n \cdot (n+1)^2$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (n+2) \cdot (3 \cdot n+5)}{12}$.
- (23) If for every n holds $s(n) = n \cdot (n+1)^2 \cdot (n+2)$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n \cdot (n+1) \cdot (n+2) \cdot (n+3) \cdot (2 \cdot n+3)}{10}$.
- (24) If for every n holds $s(n) = n \cdot (n+1) \cdot 2^n$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 2^{n+1} \cdot ((n^2 - n) + 2) - 4$.
- (25) Suppose that for every n such that $n \geq 2$ holds $s(n) = \frac{1}{(n-1) \cdot (n+1)}$ and $s(0) = 0$ and $s(1) = 0$. Let given n . If $n \geq 2$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{3}{4} - \frac{1}{2 \cdot n} - \frac{1}{2 \cdot (n+1)}$.
- (26) If for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(2 \cdot n-1) \cdot (2 \cdot n+1)}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n}{2 \cdot n+1}$.
- (27) If for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(3 \cdot n-2) \cdot (3 \cdot n+1)}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{n}{3 \cdot n+1}$.

- (28) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(2 \cdot n - 1) \cdot (2 \cdot n + 1) \cdot (2 \cdot n + 3)}$ and $s(0) = 0$. Let given n . If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{12} - \frac{1}{4 \cdot (2 \cdot n + 1) \cdot (2 \cdot n + 3)}$.
- (29) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{1}{(3 \cdot n - 2) \cdot (3 \cdot n + 1) \cdot (3 \cdot n + 4)}$ and $s(0) = 0$. Let given n . If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{24} - \frac{1}{6 \cdot (3 \cdot n + 1) \cdot (3 \cdot n + 4)}$.
- (30) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{2 \cdot n - 1}{n \cdot (n + 1) \cdot (n + 2)}$ and $s(0) = 0$. Let given n . If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = (\frac{3}{4} - \frac{2}{n + 2}) + \frac{1}{2 \cdot (n + 1) \cdot (n + 2)}$.
- (31) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{n + 2}{n \cdot (n + 1) \cdot (n + 3)}$ and $s(0) = 0$. Let given n . If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{29}{36} - \frac{1}{n + 3} - \frac{3}{2 \cdot (n + 2) \cdot (n + 3)} - \frac{4}{3 \cdot (n + 1) \cdot (n + 2) \cdot (n + 3)}$.
- (32) If for every n holds $s(n) = \frac{(n + 1) \cdot 2^n}{(n + 2) \cdot (n + 3)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{2^{n+1}}{n + 3} - \frac{1}{2}$.
- (33) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{n^2 \cdot 4^n}{(n + 1) \cdot (n + 2)}$ and $s(0) = 0$. Let given n . If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{2}{3} + \frac{(n - 1) \cdot 4^{n+1}}{3 \cdot (n + 2)}$.
- (34) If for every n such that $n \geq 1$ holds $s(n) = \frac{n + 2}{n \cdot (n + 1) \cdot 2^n}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{(n + 1) \cdot 2^n}$.
- (35) Suppose that for every n such that $n \geq 1$ holds $s(n) = \frac{2 \cdot n + 3}{n \cdot (n + 1) \cdot 3^n}$ and $s(0) = 0$. Let given n . If $n \geq 1$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{(n + 1) \cdot 3^n}$.
- (36) If for every n holds $s(n) = \frac{(-1)^n \cdot 2^{n+1}}{(2^{n+1} + (-1)^{n+1}) \cdot (2^{n+2} + (-1)^{n+2})}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{3} + \frac{(-1)^{n+2}}{3 \cdot (2^{n+2} + (-1)^{n+2})}$.
- (37) If for every n holds $s(n) = n! \cdot n$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = (n + 1)! - 1$.
- (38) If for every n holds $s(n) = \frac{n}{(n + 1)!}$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{1}{(n + 1)!}$.
- (39) If for every n such that $n \geq 1$ holds $s(n) = \frac{(n^2 + n) - 1}{(n + 2)!}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = \frac{1}{2} - \frac{n + 1}{(n + 2)!}$.
- (40) If for every n such that $n \geq 1$ holds $s(n) = \frac{n \cdot 2^n}{(n + 2)!}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) = 1 - \frac{2^{n+1}}{(n + 2)!}$.

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