

Several Differentiation Formulas of Special Functions. Part VII

Fuguo Ge
 Qingdao University of Science
 and Technology
 China

Bing Xie
 Qingdao University of Science
 and Technology
 China

Summary. In this article, we prove a series of differentiation identities [2] involving the arctan and arccot functions and specific combinations of special functions including trigonometric and exponential functions.

MML identifier: FDIFF_11, version: 7.10.01 4.111.1036

The papers [13], [15], [1], [10], [16], [5], [12], [3], [6], [9], [4], [11], [8], [14], and [7] provide the terminology and notation for this paper.

For simplicity, we adopt the following rules: x denotes a real number, n denotes an element of $\mathbb{N}$, Z denotes an open subset of $\mathbb{R}$, and f, g denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

Next we state a number of propositions:

- (1) Suppose $Z \subseteq \text{dom}((\text{the function arctan}) \cdot (\text{the function sin}))$ and for every x such that $x \in Z$ holds $-1 < \sin x < 1$. Then
 - (i) $(\text{the function arctan}) \cdot (\text{the function sin})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function arctan}) \cdot (\text{the function sin}))'_{|Z}(x) = \frac{\cos x}{1+(\sin x)^2}$.
- (2) Suppose $Z \subseteq \text{dom}((\text{the function arccot}) \cdot (\text{the function sin}))$ and for every x such that $x \in Z$ holds $-1 < \sin x < 1$. Then
 - (i) $(\text{the function arccot}) \cdot (\text{the function sin})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function arccot}) \cdot (\text{the function sin}))'_{|Z}(x) = -\frac{\cos x}{1+(\sin x)^2}$.
- (3) Suppose $Z \subseteq \text{dom}((\text{the function arctan}) \cdot (\text{the function cos}))$ and for every x such that $x \in Z$ holds $-1 < \cos x < 1$. Then

- (i) (the function $\arctan$) $\cdot$ (the function $\cos$) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot (\text{the function } \cos))'_{|Z}(x) = -\frac{\sin x}{1+(\cos x)^2}$.
- (4) Suppose $Z \subseteq \text{dom}((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \cos))$ and for every x such that $x \in Z$ holds $-1 < \cos x < 1$. Then
 - (i) (the function arccot) $\cdot$ (the function $\cos$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \cos))'_{|Z}(x) = \frac{\sin x}{1+(\cos x)^2}$.
- (5) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \tan))$ and for every x such that $x \in Z$ holds $-1 < \tan x < 1$. Then
 - (i) (the function $\arctan$) $\cdot$ (the function $\tan$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot (\text{the function } \tan))'_{|Z}(x) = 1$.
- (6) Suppose $Z \subseteq \text{dom}((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \tan))$ and for every x such that $x \in Z$ holds $-1 < \tan x < 1$. Then
 - (i) (the function arccot) $\cdot$ (the function $\tan$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \tan))'_{|Z}(x) = -1$.
- (7) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \cot))$ and for every x such that $x \in Z$ holds $-1 < \cot x < 1$. Then
 - (i) (the function $\arctan$) $\cdot$ (the function $\cot$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot (\text{the function } \cot))'_{|Z}(x) = -1$.
- (8) Suppose $Z \subseteq \text{dom}((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \cot))$ and for every x such that $x \in Z$ holds $-1 < \cot x < 1$. Then
 - (i) (the function arccot) $\cdot$ (the function $\cot$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \cot))'_{|Z}(x) = 1$.
- (9) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \arctan x < 1$. Then
 - (i) (the function $\arctan$) $\cdot$ (the function $\arctan$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot (\text{the function } \arctan))'_{|Z}(x) = \frac{1}{(1+x^2) \cdot (1+(\arctan x)^2)}$.
- (10) Suppose $Z \subseteq \text{dom}((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \arctan x < 1$. Then
 - (i) (the function arccot) $\cdot$ (the function $\arctan$) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \arctan))'_{|Z}(x) = -\frac{1}{(1+x^2) \cdot (1+(\arctan x)^2)}$.

- (11) Suppose $Z \subseteq \text{dom}((\text{the function } \arctan) \cdot (\text{the function } \operatorname{arccot}))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \operatorname{arccot} x < 1$. Then
- (i) $(\text{the function } \arctan) \cdot (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot (\text{the function } \operatorname{arccot}))'_{|Z}(x) = -\frac{1}{(1+x^2) \cdot (1+(\operatorname{arccot} x)^2)}$.
- (12) Suppose $Z \subseteq \text{dom}((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \operatorname{arccot}))$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $-1 < \operatorname{arccot} x < 1$. Then
- (i) $(\text{the function } \operatorname{arccot}) \cdot (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \operatorname{arccot}) \cdot (\text{the function } \operatorname{arccot}))'_{|Z}(x) = \frac{1}{(1+x^2) \cdot (1+(\operatorname{arccot} x)^2)}$.
- (13) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \sin) \cdot (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot (\text{the function } \arctan))'_{|Z}(x) = \frac{\cos \arctan x}{1+x^2}$.
- (14) Suppose $Z \subseteq \text{dom}((\text{the function } \sin) \cdot (\text{the function } \operatorname{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \sin) \cdot (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot (\text{the function } \operatorname{arccot}))'_{|Z}(x) = -\frac{\cos \operatorname{arccot} x}{1+x^2}$.
- (15) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) \cdot (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot (\text{the function } \arctan))'_{|Z}(x) = -\frac{\sin \arctan x}{1+x^2}$.
- (16) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) \cdot (\text{the function } \operatorname{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) \cdot (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot (\text{the function } \operatorname{arccot}))'_{|Z}(x) = \frac{\sin \operatorname{arccot} x}{1+x^2}$.
- (17) Suppose $Z \subseteq \text{dom}((\text{the function } \tan) \cdot (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \tan) \cdot (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \tan) \cdot (\text{the function } \arctan))'_{|Z}(x) = \frac{1}{(\cos \arctan x)^2 \cdot (1+x^2)}$.
- (18) Suppose $Z \subseteq \text{dom}((\text{the function } \tan) \cdot (\text{the function } \operatorname{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \tan) \cdot (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \tan) \cdot (\text{the function } \operatorname{arccot}))'_{|Z}(x) = -\frac{1}{(\cos \operatorname{arccot} x)^2 \cdot (1+x^2)}$.

- (19) Suppose $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function arctan}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cot}) \cdot (\text{the function arctan})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cot}) \cdot (\text{the function arctan}))'_{|Z}(x) = -\frac{1}{(\sin \arctan x)^2 \cdot (1+x^2)}$.
- (20) Suppose $Z \subseteq \text{dom}((\text{the function cot}) \cdot (\text{the function arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cot}) \cdot (\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cot}) \cdot (\text{the function arccot}))'_{|Z}(x) = \frac{1}{(\sin \arccot x)^2 \cdot (1+x^2)}$.
- (21) Suppose $Z \subseteq \text{dom}((\text{the function sec}) \cdot (\text{the function arctan}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function sec}) \cdot (\text{the function arctan})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function sec}) \cdot (\text{the function arctan}))'_{|Z}(x) = \frac{\sin \arctan x}{(\cos \arctan x)^2 \cdot (1+x^2)}$.
- (22) Suppose $Z \subseteq \text{dom}((\text{the function sec}) \cdot (\text{the function arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function sec}) \cdot (\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function sec}) \cdot (\text{the function arccot}))'_{|Z}(x) = -\frac{\sin \arccot x}{(\cos \arccot x)^2 \cdot (1+x^2)}$.
- (23) Suppose $Z \subseteq \text{dom}((\text{the function cosec}) \cdot (\text{the function arctan}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cosec}) \cdot (\text{the function arctan})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cosec}) \cdot (\text{the function arctan}))'_{|Z}(x) = -\frac{\cos \arctan x}{(\sin \arctan x)^2 \cdot (1+x^2)}$.
- (24) Suppose $Z \subseteq \text{dom}((\text{the function cosec}) \cdot (\text{the function arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cosec}) \cdot (\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cosec}) \cdot (\text{the function arccot}))'_{|Z}(x) = \frac{\cos \arccot x}{(\sin \arccot x)^2 \cdot (1+x^2)}$.
- (25) Suppose $Z \subseteq \text{dom}((\text{the function sin}) \cdot (\text{the function arctan}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function sin}) \cdot (\text{the function arctan})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot (\text{the function arctan}))'_{|Z}(x) = \cos x \cdot \arctan x + \frac{\sin x}{1+x^2}$.
- (26) Suppose $Z \subseteq \text{dom}((\text{the function sin}) \cdot (\text{the function arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function sin}) \cdot (\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) \cdot (\text{the function arccot}))'_{|Z}(x) = \cos x \cdot \arccot x - \frac{\sin x}{1+x^2}$.

- (27) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) (\text{the function } \arctan))'_|Z(x) = -\sin x \cdot \arctan x + \frac{\cos x}{1+x^2}$.
- (28) Suppose $Z \subseteq \text{dom}((\text{the function } \cos) (\text{the function } \text{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) (\text{the function } \text{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) (\text{the function } \text{arccot}))'_|Z(x) = -\sin x \cdot \text{arccot } x - \frac{\cos x}{1+x^2}$.
- (29) Suppose $Z \subseteq \text{dom}((\text{the function } \tan) (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \tan) (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \tan) (\text{the function } \arctan))'_|Z(x) = \frac{\arctan x}{(\cos x)^2} + \frac{\tan x}{1+x^2}$.
- (30) Suppose $Z \subseteq \text{dom}((\text{the function } \tan) (\text{the function } \text{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \tan) (\text{the function } \text{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \tan) (\text{the function } \text{arccot}))'_|Z(x) = \frac{\text{arccot } x}{(\cos x)^2} - \frac{\tan x}{1+x^2}$.
- (31) Suppose $Z \subseteq \text{dom}((\text{the function } \cot) (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cot) (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cot) (\text{the function } \arctan))'_|Z(x) = -\frac{\arctan x}{(\sin x)^2} + \frac{\cot x}{1+x^2}$.
- (32) Suppose $Z \subseteq \text{dom}((\text{the function } \cot) (\text{the function } \text{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cot) (\text{the function } \text{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \cot) (\text{the function } \text{arccot}))'_|Z(x) = -\frac{\text{arccot } x}{(\sin x)^2} - \frac{\cot x}{1+x^2}$.
- (33) Suppose $Z \subseteq \text{dom}((\text{the function } \sec) (\text{the function } \arctan))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \sec) (\text{the function } \arctan)$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sec) (\text{the function } \arctan))'_|Z(x) = \frac{\sin x \cdot \arctan x}{(\cos x)^2} + \frac{1}{\cos x \cdot (1+x^2)}$.
- (34) Suppose $Z \subseteq \text{dom}((\text{the function } \sec) (\text{the function } \text{arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \sec) (\text{the function } \text{arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sec) (\text{the function } \text{arccot}))'_|Z(x) = \frac{\sin x \cdot \text{arccot } x}{(\cos x)^2} - \frac{1}{\cos x \cdot (1+x^2)}$.

- (35) Suppose $Z \subseteq \text{dom}((\text{the function cosec}) (\text{the function arctan}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cosec}) (\text{the function arctan})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cosec}) (\text{the function arctan}))'_{|Z}(x) = -\frac{\cos x \cdot \arctan x}{(\sin x)^2} + \frac{1}{\sin x \cdot (1+x^2)}$.
- (36) Suppose $Z \subseteq \text{dom}((\text{the function cosec}) (\text{the function arccot}))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cosec}) (\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cosec}) (\text{the function arccot}))'_{|Z}(x) = -\frac{\cos x \cdot \text{arccot } x}{(\sin x)^2} - \frac{1}{\sin x \cdot (1+x^2)}$.
- (37) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function arctan})+(\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function arctan})+(\text{the function arccot}))'_{|Z}(x) = 0$.
- (38) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function arctan})-(\text{the function arccot})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function arctan})-(\text{the function arccot}))'_{|Z}(x) = \frac{2}{1+x^2}$.
- (39) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function sin}) ((\text{the function arctan})+(\text{the function arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) ((\text{the function arctan})+(\text{the function arccot})))'_{|Z}(x) = \cos x \cdot (\arctan x + \text{arccot } x)$.
- (40) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function sin}) ((\text{the function arctan})-(\text{the function arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function sin}) ((\text{the function arctan})-(\text{the function arccot})))'_{|Z}(x) = \cos x \cdot (\arctan x - \text{arccot } x) + \frac{2 \cdot \sin x}{1+x^2}$.
- (41) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cos}) ((\text{the function arctan})+(\text{the function arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cos}) ((\text{the function arctan})+(\text{the function arccot})))'_{|Z}(x) = -\sin x \cdot (\arctan x + \text{arccot } x)$.
- (42) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function cos}) ((\text{the function arctan})-(\text{the function arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function cos}) ((\text{the function arctan})-(\text{the function arccot})))'_{|Z}(x) = -\sin x \cdot (\arctan x - \text{arccot } x) + \frac{2 \cdot \cos x}{1+x^2}$.
- (43) Suppose $Z \subseteq \text{dom}(\text{the function tan})$ and $Z \subseteq]-1, 1[$. Then

- (i) (the function $\tan$) ((the function $\arctan$) + (the function arccot)) is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds ((the function $\tan$) ((the function $\arctan$) + (the function arccot)))' $\big|_Z(x) = \frac{\arctan x + \operatorname{arccot} x}{(\cos x)^2}$.
- (44) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \tan)$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function $\tan$) ((the function $\arctan$) - (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function $\tan$) ((the function $\arctan$) - (the function arccot)))' $\big|_Z(x) = \frac{\arctan x - \operatorname{arccot} x}{(\cos x)^2} + \frac{2 \cdot \tan x}{1+x^2}$.
- (45) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \cot)$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function $\cot$) ((the function $\arctan$) + (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function $\cot$) ((the function $\arctan$) + (the function arccot)))' $\big|_Z(x) = -\frac{\arctan x + \operatorname{arccot} x}{(\sin x)^2}$.
- (46) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \cot)$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function $\cot$) ((the function $\arctan$) - (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function $\cot$) ((the function $\arctan$) - (the function arccot)))' $\big|_Z(x) = -\frac{\arctan x - \operatorname{arccot} x}{(\sin x)^2} + \frac{2 \cdot \cot x}{1+x^2}$.
- (47) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \sec)$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function $\sec$) ((the function $\arctan$) + (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function $\sec$) ((the function $\arctan$) + (the function arccot)))' $\big|_Z(x) = \frac{(\arctan x + \operatorname{arccot} x) \cdot \sin x}{(\cos x)^2}$.
- (48) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \sec)$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function $\sec$) ((the function $\arctan$) - (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function $\sec$) ((the function $\arctan$) - (the function arccot)))' $\big|_Z(x) = \frac{(\arctan x - \operatorname{arccot} x) \cdot \sin x}{(\cos x)^2} + \frac{2 \cdot \sec x}{1+x^2}$.
- (49) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \operatorname{cosec})$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function cosec) ((the function $\arctan$) + (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function cosec) ((the function $\arctan$) + (the function arccot)))' $\big|_Z(x) = -\frac{(\arctan x + \operatorname{arccot} x) \cdot \cos x}{(\sin x)^2}$.
- (50) Suppose $Z \subseteq \operatorname{dom}(\text{the function } \operatorname{cosec})$ and $Z \subseteq]-1, 1[$. Then
 - (i) (the function cosec) ((the function $\arctan$) - (the function arccot)) is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds ((the function cosec) ((the function $\arctan$) - (the function arccot)))' $\big|_Z(x) = -\frac{(\arctan x - \operatorname{arccot} x) \cdot \cos x}{(\sin x)^2} + \frac{2 \cdot \operatorname{cosec} x}{1+x^2}$.
- (51) Suppose $Z \subseteq]-1, 1[$. Then

- (i) (the function $\exp$) $((\text{the function } \arctan) + (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))'_{|Z}(x) = \exp x \cdot (\arctan x + \operatorname{arccot} x)$.
- (52) Suppose $Z \subseteq]-1, 1[$. Then
- (i) (the function $\exp$) $((\text{the function } \arctan) - (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))'_{|Z}(x) = \exp x \cdot (\arctan x - \operatorname{arccot} x) + \frac{2 \cdot \exp x}{1+x^2}$.
- (53) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $\frac{(\text{the function } \arctan) + (\text{the function } \operatorname{arccot})}{\text{the function } \exp}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function } \arctan) + (\text{the function } \operatorname{arccot})}{\text{the function } \exp})'_{|Z}(x) = -\frac{\arctan x + \operatorname{arccot} x}{\exp x}$.
- (54) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $\frac{(\text{the function } \arctan) - (\text{the function } \operatorname{arccot})}{\text{the function } \exp}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{(\text{the function } \arctan) - (\text{the function } \operatorname{arccot})}{\text{the function } \exp})'_{|Z}(x) = \frac{(\frac{2}{1+x^2} - \arctan x) + \operatorname{arccot} x}{\exp x}$.
- (55) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \exp) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function $\exp$) $\cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))'_{|Z}(x) = 0$.
- (56) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \exp) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function $\exp$) $\cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \exp) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))'_{|Z}(x) = \frac{2 \cdot \exp(\arctan x - \operatorname{arccot} x)}{1+x^2}$.
- (57) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \sin) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function $\sin$) $\cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))'_{|Z}(x) = 0$.
- (58) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \sin) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) (the function $\sin$) $\cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $((\text{the function } \sin) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))'_{|Z}(x) = \frac{2 \cdot \cos(\arctan x - \operatorname{arccot} x)}{1+x^2}$.
- (59) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \cos) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \arctan) + (\text{the function } \operatorname{arccot})))'_{|Z}(x) = 0$.
- (60) Suppose $Z \subseteq \operatorname{dom}((\text{the function } \cos) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))$ and $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \cos) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot}))$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \cos) \cdot ((\text{the function } \arctan) - (\text{the function } \operatorname{arccot})))'_{|Z}(x) = -\frac{2 \cdot \sin(\arctan x - \operatorname{arccot} x)}{1+x^2}$.
- (61) Suppose $Z \subseteq]-1, 1[$. Then
- (i) $(\text{the function } \arctan) (\text{the function } \operatorname{arccot})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $((\text{the function } \arctan) (\text{the function } \operatorname{arccot}))'_{|Z}(x) = \frac{\operatorname{arccot} x - \arctan x}{1+x^2}$.
- (62) Suppose that
- (i) $Z \subseteq \operatorname{dom}(((\text{the function } \arctan) \cdot \frac{1}{f}) ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))$, and
- (ii) for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$.
- Then
- (iii) $((\text{the function } \arctan) \cdot \frac{1}{f}) ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})$ is differentiable on Z , and
- (iv) for every x such that $x \in Z$ holds $((\text{the function } \arctan) \cdot \frac{1}{f}) ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})'_{|Z}(x) = \frac{\arctan(\frac{1}{x}) - \operatorname{arccot}(\frac{1}{x})}{1+x^2}$.
- (63) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z ((\text{the function } \arctan) \cdot \frac{1}{f}))$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $\operatorname{id}_Z ((\text{the function } \arctan) \cdot \frac{1}{f})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\operatorname{id}_Z ((\text{the function } \arctan) \cdot \frac{1}{f}))'_{|Z}(x) = \arctan(\frac{1}{x}) - \frac{x}{1+x^2}$.
- (64) Suppose $Z \subseteq \operatorname{dom}(\operatorname{id}_Z ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $\operatorname{id}_Z ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})$ is differentiable on Z , and
- (ii) for every x such that $x \in Z$ holds $(\operatorname{id}_Z ((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))'_{|Z}(x) = \operatorname{arccot}(\frac{1}{x}) + \frac{x}{1+x^2}$.
- (65) Suppose $Z \subseteq \operatorname{dom}(g ((\text{the function } \arctan) \cdot \frac{1}{f}))$ and $g = \square^2$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
- (i) $g ((\text{the function } \arctan) \cdot \frac{1}{f})$ is differentiable on Z , and

- (ii) for every x such that $x \in Z$ holds $(g((\text{the function } \arctan) \cdot \frac{1}{f}))'_{|Z}(x) = 2 \cdot x \cdot \arctan(\frac{1}{x}) - \frac{x^2}{1+x^2}$.
- (66) Suppose $Z \subseteq \text{dom}(g((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))$ and $g = \square^2$ and for every x such that $x \in Z$ holds $f(x) = x$ and $-1 < (\frac{1}{f})(x) < 1$. Then
 - (i) $g((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f})$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(g((\text{the function } \operatorname{arccot}) \cdot \frac{1}{f}))'_{|Z}(x) = 2 \cdot x \cdot \operatorname{arccot}(\frac{1}{x}) + \frac{x^2}{1+x^2}$.
- (67) Suppose $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $(\text{the function } \arctan)(x) \neq 0$. Then
 - (i) $\frac{1}{\text{the function } \arctan}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function } \arctan})'_{|Z}(x) = -\frac{1}{(\arctan x)^2 \cdot (1+x^2)}$.
- (68) Suppose $Z \subseteq]-1, 1[$. Then
 - (i) $\frac{1}{\text{the function } \operatorname{arccot}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function } \operatorname{arccot}})'_{|Z}(x) = \frac{1}{(\operatorname{arccot} x)^2 \cdot (1+x^2)}$.

One can prove the following propositions:

- (69) Suppose $Z \subseteq \text{dom}(\frac{1}{n(\text{the function } \arctan)^n})$ and $Z \subseteq]-1, 1[$ and $n > 0$ and for every x such that $x \in Z$ holds $\arctan x \neq 0$. Then
 - (i) $\frac{1}{n(\text{the function } \arctan)^n}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{n(\text{the function } \arctan)^n})'_{|Z}(x) = -\frac{1}{((\arctan x)^{n+1}) \cdot (1+x^2)}$.
- (70) Suppose $Z \subseteq \text{dom}(\frac{1}{n(\text{the function } \operatorname{arccot})^n})$ and $Z \subseteq]-1, 1[$ and $n > 0$. Then
 - (i) $\frac{1}{n(\text{the function } \operatorname{arccot})^n}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{1}{n(\text{the function } \operatorname{arccot})^n})'_{|Z}(x) = \frac{1}{((\operatorname{arccot} x)^{n+1}) \cdot (1+x^2)}$.
- (71) Suppose $Z \subseteq \text{dom}(2(\text{the function } \arctan)^{\frac{1}{2}})$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $\arctan x > 0$. Then
 - (i) $2(\text{the function } \arctan)^{\frac{1}{2}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(2(\text{the function } \arctan)^{\frac{1}{2}})'_{|Z}(x) = \frac{(\arctan x)^{-\frac{1}{2}}}{1+x^2}$.
- (72) Suppose $Z \subseteq \text{dom}(2(\text{the function } \operatorname{arccot})^{\frac{1}{2}})$ and $Z \subseteq]-1, 1[$. Then
 - (i) $2(\text{the function } \operatorname{arccot})^{\frac{1}{2}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(2(\text{the function } \operatorname{arccot})^{\frac{1}{2}})'_{|Z}(x) = -\frac{(\operatorname{arccot} x)^{-\frac{1}{2}}}{1+x^2}$.

- (73) Suppose $Z \subseteq \text{dom}(\frac{2}{3}(\text{the function } \arctan)^{\frac{3}{2}})$ and $Z \subseteq]-1, 1[$ and for every x such that $x \in Z$ holds $\arctan x > 0$. Then
- (i) $\frac{2}{3}(\text{the function } \arctan)^{\frac{3}{2}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{2}{3}(\text{the function } \arctan)^{\frac{3}{2}})'_Z(x) = \frac{(\arctan x)^{\frac{1}{2}}}{1+x^2}$.
- (74) Suppose $Z \subseteq \text{dom}(\frac{2}{3}(\text{the function } \text{arccot})^{\frac{3}{2}})$ and $Z \subseteq]-1, 1[$. Then
- (i) $\frac{2}{3}(\text{the function } \text{arccot})^{\frac{3}{2}}$ is differentiable on Z , and
 - (ii) for every x such that $x \in Z$ holds $(\frac{2}{3}(\text{the function } \text{arccot})^{\frac{3}{2}})'_Z(x) = -\frac{(\text{arccot } x)^{\frac{1}{2}}}{1+x^2}$.

REFERENCES

- [1] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(2):357–367, 1990.
- [2] Fritz Chemnitz. *Differentiation und Integration ausgewählter Beispiele*. VEB Verlag Technik, Berlin, 1956.
- [3] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(1):35–40, 1990.
- [4] Jarosław Kotowicz. Partial functions from a domain to a domain. *Formalized Mathematics*, 1(4):697–702, 1990.
- [5] Jarosław Kotowicz. Partial functions from a domain to the set of real numbers. *Formalized Mathematics*, 1(4):703–709, 1990.
- [6] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(2):269–272, 1990.
- [7] Xiquan Liang and Bing Xie. Inverse trigonometric functions $\arctan$ and arccot . *Formalized Mathematics*, 16(2):147–158, 2008.
- [8] Konrad Raczkowski. Integer and rational exponents. *Formalized Mathematics*, 2(1):125–130, 1991.
- [9] Konrad Raczkowski and Paweł Sadowski. Real function differentiability. *Formalized Mathematics*, 1(4):797–801, 1990.
- [10] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(4):777–780, 1990.
- [11] Yasunari Shidama. The Taylor expansions. *Formalized Mathematics*, 12(2):195–200, 2004.
- [12] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(3):445–449, 1990.
- [13] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(1):67–71, 1990.
- [14] Peng Wang and Bo Li. Several differentiation formulas of special functions. Part V. *Formalized Mathematics*, 15(3):73–79, 2007.
- [15] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(1):181–186, 1990.
- [16] Yuguang Yang and Yasunari Shidama. Trigonometric functions and existence of circle ratio. *Formalized Mathematics*, 7(2):255–263, 1998.

Received September 23, 2008
