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Regular Expression Quantifiers – at least m Occurrences

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Summary. This is the second article on regular expression quantifiers. [4] introduced the quantifiers m to n occurrences and optional occurrence. In the sequel, the quantifiers: at least m occurrences and positive closure (at least 1 occurrence) are introduced. Notation and terminology were taken from [8], several properties of regular expressions from [7].

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The notation and terminology used here are introduced in the following papers: [5], [1], [6], [2], [3], and [4].

1. Preliminaries

For simplicity, we follow the rules: E, x denote sets, A, B, C denote subsets of E^ω, a, b denote elements of E^ω, and k, l, m, n denote natural numbers.

The following proposition is true

(1) If $B \subseteq A^*$, then $(A^*) \frown B \subseteq A^*$ and $B \frown A^* \subseteq A^*$.

2. At least m Occurrences

Let us consider E, A, n. The functor $A^{n,\cdots}$ yielding a subset of E^ω is defined as follows:

(Def. 1) $A^{n,\cdots} = \bigcup\{B : \bigvee_m (n \leq m \ \wedge \ B = A^m)\}$.

We now state a number of propositions:

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(2) $x \in A^{n,\cdots}$ iff there exists m such that $n \leq m$ and $x \in A^m$.
(3) If $n \leq m$, then $A^m \subseteq A^{n,\cdots}$.
(4) $A^{n,\cdots} = \emptyset$ iff $n > 0$ and $A = \emptyset$.
(5) If $m \leq n$, then $A^{n,\cdots} \subseteq A^{m,\cdots}$.
(6) If $k \leq m$, then $A^{m,n} \subseteq A^{k,\cdots}$.
(7) If $m \leq n+1$, then $A^{m,n} \cup (A^{(n+1),\cdots}) = A^{m,\cdots}$.
(8) $A^n \cup (A^{(n+1),\cdots}) = A^{n,\cdots}$.
(9) $A^{n,\cdots} \subseteq A^*$.
(10) $\langle\rangle_E \in A^{n,\cdots}$ iff $n = 0$ or $\langle\rangle_E \in A$.
(11) $A^{n,\cdots} = A^*$ iff $\langle\rangle_E \in A$ or $n = 0$.
(12) $A^* = A^{0,n} \cup (A^{(n+1),\cdots})$.
(13) If $A \subseteq B$, then $A^{n,\cdots} \subseteq B^{n,\cdots}$.
(14) If $x \in A$ and $x \neq \langle\rangle_E$, then $A^{n,\cdots} \neq \{\langle\rangle_E\}$.
(15) $A^{n,\cdots} = \{\langle\rangle_E\}$ iff $A = \{\langle\rangle_E\}$ or $n = 0$ and $A = \emptyset$.
(16) $A^{(n+1),\cdots} = (A^{n,\cdots}) \frown A$.
(17) $(A^{m,\cdots}) \frown A^* = A^{m,\cdots}$.
(18) $(A^{m,\cdots}) \frown (A^{n,\cdots}) = A^{(m+n),\cdots}$.
(19) If $n > 0$, then $(A^{m,\cdots})^n = A^{m \cdot n,\cdots}$.
(20) $(A^{n,\cdots})^* = (A^{n,\cdots})?$.
(21) If $A \subseteq C^{m,\cdots}$ and $B \subseteq C^{n,\cdots}$, then $A \frown B \subseteq C^{(m+n),\cdots}$.
(22) $A^{(n+k),\cdots} = (A^{n,\cdots}) \frown A^k$.
(23) $A \frown (A^{n,\cdots}) = (A^{n,\cdots}) \frown A$.
(24) $(A^k) \frown (A^{n,\cdots}) = (A^{n,\cdots}) \frown A^k$.
(25) $(A^{k,l}) \frown (A^{n,\cdots}) = (A^{n,\cdots}) \frown A^{k,l}$.
(26) If $\langle\rangle_E \in B$, then $A \subseteq A \frown (B^{n,\cdots})$ and $A \subseteq (B^{n,\cdots}) \frown A$.
(27) $(A^{m,\cdots}) \frown (A^{n,\cdots}) = (A^{n,\cdots}) \frown (A^{m,\cdots})$.
(28) If $A \subseteq B^{k,\cdots}$ and $n > 0$, then $A^n \subseteq B^{k,\cdots}$.
(29) If $A \subseteq B^{k,\cdots}$ and $n > 0$, then $A^{n,\cdots} \subseteq B^{k,\cdots}$.
(30) $(A^*) \frown A = A^{1,\cdots}$.
(31) $(A^*) \frown A^k = A^{k,\cdots}$.
(32) $(A^{m,\cdots}) \frown A^* = (A^*) \frown (A^{m,\cdots})$.
(33) If $k \leq l$, then $(A^{n,\cdots}) \frown A^{k,l} = A^{(n+k),\cdots}$.
(34) If $k \leq l$, then $(A^*) \frown A^{k,l} = A^{k,\cdots}$.
(35) $A^{mn,\cdots} \subseteq A^{m \cdot n,\cdots}$.
(36) $A^{mn,\cdots} \subseteq (A^{n,\cdots})^m$.
(37) If $a \in C^{m,\cdots}$ and $b \in C^{n,\cdots}$, then $a \frown b \in C^{(m+n),\cdots}$.
(38) If $A^{k,\cdots} = \{x\}$, then $x = \langle\rangle_E$.

(39) If $A \subseteq B^*$, then $A^{n,\cdots} \subseteq B^*$.

(40) $A? \subseteq A^{k,\cdots}$ iff $k = 0$ or $\langle\rangle_E \in A$.

(41) $(A^{k,\cdots}) \frown A? = A^{k,\cdots}$.

(42) $(A^{k,\cdots}) \frown A? = A? \frown (A^{k,\cdots})$.

(43) If $B \subseteq A^*$, then $(A^{k,\cdots}) \frown B \subseteq A^{k,\cdots}$ and $B \frown (A^{k,\cdots}) \subseteq A^{k,\cdots}$.

(44) $A \cap B^{k,\cdots} \subseteq (A^{k,\cdots}) \cap (B^{k,\cdots})$.

(45) $(A^{k,\cdots}) \cup (B^{k,\cdots}) \subseteq (A \cup B)^{k,\cdots}$.

(46) $\langle x\rangle \in A^{k,\cdots}$ iff $\langle x\rangle \in A$ but $\langle\rangle_E \in A$ or $k \leq 1$.

(47) If $A \subseteq B^{k,\cdots}$, then $B^{k,\cdots} = (B \cup A)^{k,\cdots}$.

3. Positive Closure

Let us consider E, A. The functor A^+ yielding a subset of E^ω is defined as follows:

(Def. 2) $A^+ = \bigcup\{B : \bigvee_n \ (n > 0 \ \wedge \ B = A^n)\}$.

Next we state a number of propositions:

(48) $x \in A^+$ iff there exists n such that $n > 0$ and $x \in A^n$.

(49) If $n > 0$, then $A^n \subseteq A^+$.

(50) $A^+ = A^{1,\cdots}$.

(51) $A^+ = \emptyset$ iff $A = \emptyset$.

(52) $A^+ = (A^*) \frown A$.

(53) $A^* = \{\langle\rangle_E\} \cup A^+$.

(54) $A^+ = A^{1,n} \cup (A^{(n+1),\cdots})$.

(55) $A^+ \subseteq A^*$.

(56) $\langle\rangle_E \in A^+$ iff $\langle\rangle_E \in A$.

(57) $A^+ = A^*$ iff $\langle\rangle_E \in A$.

(58) If $A \subseteq B$, then $A^+ \subseteq B^+$.

(59) $A \subseteq A^+$.

(60) $A^{*+} = A^*$ and $A^{+*} = A^*$.

(61) If $A \subseteq B^*$, then $A^+ \subseteq B^*$.

(62) $A^{++} = A^+$.

(63) If $x \in A$ and $x \neq \langle\rangle_E$, then $A^+ \neq \{\langle\rangle_E\}$.

(64) $A^+ = \{\langle\rangle_E\}$ iff $A = \{\langle\rangle_E\}$.

(65) $A^+? = A^*$ and $A?^+ = A^*$.

(66) If a, $b \in C^+$, then $a \frown b \in C^+$.

(67) If $A \subseteq C^+$ and $B \subseteq C^+$, then $A \frown B \subseteq C^+$.

(68) $A \frown A \subseteq A^+$.

(69) If $A^+ = \{x\}$, then $x = \langle\rangle_E$.
(70) $A \frown A^+ = A^+ \frown A$.
(71) $(A^k) \frown A^+ = A^+ \frown A^k$.
(72) $(A^{m,n}) \frown A^+ = A^+ \frown A^{m,n}$.
(73) If $\langle\rangle_E \in B$, then $A \subseteq A \frown B^+$ and $A \subseteq B^+ \frown A$.
(74) $A^+ \frown A^+ = A^{2,\cdots}$.
(75) $A^+ \frown A^k = A^{(k+1),\cdots}$.
(76) $A^+ \frown A = A^{2,\cdots}$.
(77) If $k \leq l$, then $A^+ \frown A^{k,l} = A^{(k+1),\cdots}$.
(78) If $A \subseteq B^+$ and $n > 0$, then $A^n \subseteq B^+$.
(79) $A^+ \frown A? = A? \frown A^+$.
(80) $A^+ \frown A? = A^+$.
(81) $A? \subseteq A^+$ iff $\langle\rangle_E \in A$.
(82) If $A \subseteq B^+$, then $A^+ \subseteq B^+$.
(83) If $A \subseteq B^+$, then $B^+ = (B \cup A)^+$.
(84) If $n > 0$, then $A^{n,\cdots} \subseteq A^+$.
(85) If $m > 0$, then $A^{m,n} \subseteq A^+$.
(86) $(A^*) \frown A^+ = A^+ \frown A^*$.
(87) $A^{+k} \subseteq A^{k,\cdots}$.
(88) $A^{+m,n} \subseteq A^{m,\cdots}$.
(89) If $A \subseteq B^+$ and $n > 0$, then $A^{n,\cdots} \subseteq B^+$.
(90) $A^+ \frown (A^{k,\cdots}) = A^{(k+1),\cdots}$.
(91) $A^+ \frown (A^{k,\cdots}) = (A^{k,\cdots}) \frown A^+$.
(92) $A^+ \frown A^* = A^+$.
(93) If $B \subseteq A^*$, then $A^+ \frown B \subseteq A^+$ and $B \frown A^+ \subseteq A^+$.
(94) $(A \cap B)^+ \subseteq A^+ \cap B^+$.
(95) $A^+ \cup B^+ \subseteq (A \cup B)^+$.
(96) $\langle x\rangle \in A^+$ iff $\langle x\rangle \in A$.

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