Nina Siemieniuk
University of Bialystok
e-mail: n.siemieniuk@uwb.edu.pl
ORCID: 0000-0002-8819-1301

Tomasz Siemieniuk
Eastern European Academy
of Applied Sciences in Bialystok
e-mail: tomasz.siemieniuk@wans.edu.pl
ORCID: 0000-0003-4776-3323

DOI: 10.15290/oes.2025.02.120.13

THE ANALYSIS OF THE USE OF DETERMINISTIC CHAOS METHODS AS AN EARLY WARNING SYSTEM FOR BANKRUPTCY[1]

Summary

Purpose | The main objective of the publication is to present selected issues related to early warning systems of enterprises based on intelligent systems.

Research method | The study used the theory of deterministic chaos as one of the innovative methods of creating an early warning system, which is an advantage over classic methods of analysis based on linear systems.

Results | Charts of companies listed on the Polish stock exchange indicate that these are not straight lines, but zigzag lines, which are assigned to non-linear systems. Research shows that it is possible to create an early warning system based on deterministic chaos theory as a non-linear system.

[1] The article is financed by the Faculty of Management as part of the International Scientific Conference "Management in Economy 4.0". The article has been supported by the Ministry of Science and Higher Education as part of a grant to maintain the research potential of the Faculty of Management of the University of Bialystok.
The article was received on 8.10.2024, accepted on 12.05.2025.

Originality/value/implications/recommendations | The results show that deterministic chaos theory can be an important complement to the classical methods of the early warning system.

Keywords: deterministic chaos, early warning system, listed companies

JEL classification: G, G1, G17

1. Introduction

In the economy, we can notice complex phenomena characterised by the dynamics of change, not subject to the simple laws of mathematical statistics and to inexplicable by the laws of economics. The description of this type of studied phenomena includes a large number of specific variables that have a non-linear structure with complex relationships between these variables (multidimensionality and non-linearity of systems). For many years, attempts have been made to develop methods and discover relationships between individual variables affecting the behavior of the system. These attempts usually ended with the creation of complex structures or systems of equations, often unsolvable due to their complicated structure. The possibility of using computers for multiple iterations of structures described by differential systems and the development of mathematics resulted in the transfer of statistical models to the plane of new scientific disciplines – chaos theory and fractal analysis. In chaos theory, it is assumed that the complexity of phenomena may be the cause of significantly different behavior in time of systems (exponential divergence of trajectories in phase space) with similar phase portraits in the initial phases of the cycle. Dynamical phenomena can therefore be described by dynamical models, the irregularity of which depends on the degree of their nonlinearity [Siemieniuk, 2001, p. 97].

The article presents selected issues related to early warning systems of enterprises based on intelligent systems. The authors indicate the possibility of creating an early warning system based on the theory of deterministic chaos. The system created in this way would provide managers at the strategic level with information about emerging threats and opportunities in the company's

environment. The main aim of the publication is to analyse the use of deterministic chaos methods as an early warning system for enterprises against bankruptcy on the example of the selected companies listed on the Polish stock exchange.

2. Selected early warning models

As T. Korol writes, "the current global financial crisis has proven that even the best international corporations should constantly monitor their financial situation and the financial situation of the companies they work with" [Korol, 2010, p. 12]. As K. Dębkowska points out in her work, in relation to a listed company, conducting a full analysis of the financial condition is undoubtedly very laborious, which is why models of forecasting the economic condition of the company were used to solve this problem [Dębkowska, 2012, p. 175]. As E. Piątek and M. Konat-Staniek write: "Economic literature includes ratio analysis, discriminant analysis used in 30% of cases, logit analysis (in 21%), neural networks (9%), and less frequently used expert systems, chaos theory" [Aziz, Dar, 2004], among the most frequently used tools for assessing the risk of insolvency and, consequently, bankruptcy (bankruptcy). Below, there is an overview and characteristics of the selected methods of assessing the economic condition of an enterprise: ratio analysis, discriminant analysis, logit models, artificial neural networks.

Ratio analysis is an extension of the initial analysis of financial statements. On the basis of the reports, it is possible to calculate indicators that can be used to characterise various aspects of the economic activity of a given company. As N. Siemieniuk writes: "Financial analysis deals with methods, procedures and techniques of research proceedings and the creation of rules for using them in the study of phenomena belonging to the financial management in the enterprise. The results of such an analysis have a high cognitive and usable value for both the company's management and stock market investors. The management of the company uses the results of the analysis to manage financial management and to choose a strategy for running current operations" [Siemieniuk, 2001, pp. 54–55]. Ratio analysis, in turn, allows you to determine the financial capabilities of a given business entity.

With the help of ratio analysis, we can examine various areas of the company's activity [Waśniewski, Skoczylas, 2004, p. 357]. Over the years, a huge number of indicators have been created in finance. The most common and most frequently used ones include [Sierpińska, Jachna, 1997, p. 206]:

1. financial liquidity ratio – measures the ability of the company to meet its short-term liabilities;
2. efficiency ratio (asset utilisation) – shows how the company converts its receivables and inventories into cash;
3. capital adequacy ratio (debt) – tells about the company's debt and its ability to liquidate the resulting debt;
4. profitability ratio – evaluates what profits a given company achieves from the incurred expenditures;
5. stock indices – assess the market value of a company, which depends on the price of its shares;
6. cash flow indicators.

Ratio analysis allows for an objective assessment of the economic and financial situation of a given business entity. Financial ratios make it possible to assess: the profitability of the company, the size of the assets held, the sources of its coverage, the degree of financial liquidity and the ability to service the debt [Bień, 2011, p. 167].

On the other hand, the discriminant analysis is a summary of the financial analysis of the company, classifying the examined company at a certain level. As D. Letkowski points out, "discriminatory models are a discriminatory function that allows for a synthetic measure of assessing the financial risk of an enterprise" [Letkowski, 2014, p. 316]. According to D. Letkowski, the most well-known and most popular model for assessing the poor economic condition of enterprises is the Altman Model [Letkowski, 2014, p. 316]. Altman was a forerunner in the use of multivariate discriminant analysis in research on the economic condition of enterprises. In 1968, his first model appeared, to which Altman used a sample consisting of 66 companies. Half of the selected manufacturing companies were in good economic condition, and the others were in impending bankruptcy. This sample was examined using 22 indicators, of which 5 were selected for the model [Altman, 2000, pp. 6–8]. Using

a multidimensional discriminant analysis in research on the economic condition of enterprises, appropriate coefficients were selected for the selected indicators. The final version of the model took the following form [Gołaszewski, et al., 2001, p. 63]:

$$X = 1.2\,X1 + 1.4\,X2 + 3.3\,X3 + 0.6\,X4 + 0.999\,X5$$

Legend:

X – denotes the total score,
X1 – means working capital defined as the difference between assets and current liabilities,
X2 – means the profitability of the company's accumulated retained earnings,
X3 – is the EBIT/asset quotient,
X4 – is the quotient of the market value of the company/the accounting value of the debt,
X5 – is the quotient of sales revenues/assets.

The Altman model is one of the most popular tools for examining the economic condition of enterprises. A broader discussion on the Altman model in the aspect of early warning of enterprises is written by [Stabryła, 2002, pp. 123–124, 359; Wajda, 2009, pp. 29–34; Jędrzejewski, 2005, pp. 91–92].

After modifying the model of the bankruptcy threat of E. Altman, the new formula is as follows [*Insurance in the Face...*, 2014, p.166]:

$$Z\ \text{score} = W1X_1 + W2X_2 \ldots\ldots \text{GE X n}$$

Legend:

W0 – denotes a constant parameter,
Wi – specifies the coefficients of the discriminant function for i = 1:2,....,ff,
Xi – specifies the explanatory variables of the financial ratios for i = 1.2,....,n.

Popular models in discriminant analysis developed by Polish researchers include [Bombiak, 2010, p. 145]: the model of E. Mączyńska, the Poznań model and the model of J. Gajdka and S. Stos.

E. Mączyńska's model was developed on the basis of the results of an analysis of several thousand enterprises. In this model, the group of ratios used in the model construction is dominated by the ratios of efficiency of asset use and profitability of sales revenues. Significant attention in the model is assigned to the assessment of creditworthiness, which determines the possibility of settling liabilities from the cash surplus. Inventory involvement ratios, asset productivity and debt ratio play a smaller role in this case [Bombiak, 2010, p. 146].

The Poznań model was developed by M. Hamrol, B. Czajka and M. Piechocki on the basis of the audit of financial statements of 100 Polish commercial law companies. The model assemblies were based on a constant size and four financial ratios [Hamrol et al., 2004, pp. 35–39].

The above model was written on the basis of the following relation [www 1]:

$$FD = 3{,}562*X_1 + {}_{1{,}588*X2} + 4{,}288*X3 + 6{,}719*X4 - 2{,}368$$

Legend:

X1 – net profit/total assets,
X2 – current assets – short-term inventories/liabilities,
X3 – fixed capital/total assets,
X4 – financial result on sales/sales revenues.

The model of J. Gajdka and S. Stos is one of the first models developed for the conditions of functioning of Polish enterprises [*Restructuring in the Process...*, 1996, pp. 59–63].

The construction of the model is expressed by the following formula [www 1]:

$$Z = 0.7732059 - 0.0856425*X1 + 0.0007747*X2 + \\ + 0.9220985*X3 + 0.6535995*X4 - 0.594687*X5$$

Legend:

X1 – is the quotient of sales revenues/total assets,
X2 – is the quotient of current liabilities/cost of production sold multiplied by 365,

X3 – is the quotient of net profit/total assets,
X4 – is the quotient of gross profit / net sales revenue,
X5 – is the quotient of total liabilities to total assets.

Another model of early warning against bankruptcy is the log model, which is based on the company's financial ratios [Grochowina, 2014, pp. 188–189]. Logit models are classified as classic binary classification models, in which the explanatory variable takes only two values. When examining the economic condition of an enterprise, this model analyses the impact of explanatory variables, which may be qualitative or quantitative features, on the explanatory variable of a qualitative nature, assuming a logistic analytical form of the intensity of individual explanatory variables [*Statistical Methods...*, 2014, p. 22]. To estimate the values of parameters in the logit model, the method of maximum likelihood or the weighted method of least squares is used [Ptak-Chmielewska, 2012, p. 163]. In this model, the values of the explanatory variable indicate whether or not the event that we want to predict will occur. The probability function, when the dependent variable is a dichotomous variable, is reduced to a logistic function (logistic regression, logistic model) and takes the form expressed by the formula [Kisielińska, Waszkowski, 2010, p. 19]:

$$P(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}.$$

Legend:

$X = a0+a1X1+ a2X2+.....+ a_{kXk}$,
$a0, a1,.....,a_k$ – denote the values of the coefficient estimation.

To sum up, it can be stated that logit models are a relatively simple tool for examining the economic condition of enterprises. They can be used to build early warning systems when there is a threat of business interruption for companies of all sizes. Logit models are characterised by high classification accuracy and a tendency to reduce explanatory variables, because its purpose is not to explain the effect of explanatory variables on the explanatory variable, but the accuracy of classification [Andrasik, 2005, p. 22].

As the literature on the subject indicates, mathematical, biological and computer sciences have contributed to the creation of artificial neural networks (ANNs). One of the main applications of artificial neural networks is to warn businesses against bankruptcy. The main advantages of artificial neural networks include [Grabowski, 1997, pp. 52–54]:

1. Non-linearity and non-parametric,
2. Adaptability,
3. Parallelism
4. Universality.

The numerous advantages of artificial neural networks, their high usefulness and effectiveness make their use in economic research more and more popular. Artificial neural networks have a wide range of applications in the field of broadly understood economics, ranging from: e.g. market segmentation, forecasting exchange rates, corporate bankruptcies, interest rates, budgeting issues, to the area of management (e.g. risk management model [Bose, Mohapatra, 2001, pp. 211–224].

According to the literature on the subject, after achieving market success, companies fall into the trap of enthusiasm and are not able to objectively assess the prevailing situation. Lack of action on the part of the management board or improperly taken remedial steps may contribute to the occurrence of symptoms of improper changes in the functioning of the company [Weitzel, Johnson, 1999, p. 46].

As it follows from the above, the study of the economic condition of an enterprise is a complicated process, having its basis in economic, social and legal sciences, which is why it aroused the interest of the authors of the publication.

3. Deterministic chaos – a warning system against bankruptcy

Since the late 1980s, there has been a debate about whether there is chaos (based on non-linear dynamics) in the economic or financial time series that can be used as an early warning system for bankruptcy [Peters, 1997; Orzeszko,

2005; Dockner, Prskawetz, Feichtinger, 1997; Buła 2013; 2017; 2019]. On the other hand, solutions in this field derived from chaos theory are only just emerging. Therefore, it can be concluded that chaos theory is one of the newer concepts of early warning systems. It is an attempt to move away from the idea of efficiency towards a more universal view of the mechanisms governing business management. This is reflected in the description by means of dynamic non-linear systems. This is an advantage over other methods that describe the functioning of enterprises as a linear system. In recent years, it has become evident that the phenomenon of deterministic chaos is common in a number of fields of activity and has far-reaching consequences in many disciplines of science [www 2].

The publication is an attempt at a new approach to the assessment of the economic condition of companies operating on the Warsaw Stock Exchange. The results of the analysis for the Polish stock exchange obtained by the authors will be one of the ways to fill the gap in the use of chaos theory as an early warning system against bankruptcy. Developing the above issue, the publication attempts to answer the question of what coefficients in the field of chaos theory may indicate the future bankruptcy of a company the condition of which is represented by a time series of stock exchange quotations of its shares. The following deterministic chaos measures were used for the research: fractal dimension, correlational dimension, Hurst exponents.

The fractal dimension is important information about the system because it allows to determine the minimum number of dynamic variables needed to describe the functioning of a given listed company. The smaller the number, the less complicated and more predictable it is to function on the stock exchange [Siemieniuk, 2001, p. 149]. R. Buła writes "the statement that instruments with a higher fractal dimension are assigned a higher level of risk should be accompanied by the condition that it is true only when the length of the investment horizon decreases to zero" [Buła, 2013, p. 465].

Analysing the data contained in Table 1, it can be concluded that companies with a good economic condition have a lower correlation dimension, and thus a smaller number of dynamic variables necessary for the analysis of a given company on the stock exchange, compared to companies that went bankrupt on the Warsaw Stock Exchange.

TABLE 1
Correlation and fractal dimension for selected Polish joint-stock companies

Company name	Fractal dimension	Correlation dimension (number of variables)
KGHM	**4,28**	**5**
mBank	**4,29**	**5**
PZU	**4,75**	**5**
Toora	5,23	6
Tonsil	5,94	6
Pronox	6,77	7
Loom	8,60	9

Source: the authors' own study.

In the literature on the subject, it was indicated that if the Hurst exponent (H) is smaller, the more rough (more random) the series is. A higher value of the Hurst exponent (H) means that investing in a given asset is subject to lower risk, because there is less "noise" in such a series [Peters, 1997; Siemieniuk, 2001]. This relationship was analysed on the example of the selected joint-stock companies from the Polish stock exchange (Table 2).

TABLE 2
R/S analysis for the selected Polish joint-stock companies

Name of the joint-stock company	Hurst exponent value
PZU	0.739
mBank	0.727
KGHM	0.696
Loom	0.588
Pronox	0.586
Toora	0.577
Tonsil	0.573

Source: the authors' own study.

Analysing the data from Table 2, it can be concluded that the assumptions in the literature have been confirmed in the conducted research, i.e. companies with a good economic condition have a higher Hurst exponent compared to companies that went bankrupt on the Warsaw Stock Exchange. It can be concluded that the theory of deterministic chaos can complement the classic systems of warning enterprises against bankruptcy.

4. Conclusions

This publication is an attempt at a new approach to the use of deterministic chaos theory as an early warning system against company bankruptcy. The results of the analyses obtained by the authors are one of the ways to fill the gap in this area. The directions of application of deterministic chaos theory presented in the paper probably do not exhaust all the possibilities in this respect. These considerations are of a future-oriented nature, hence the issue of their debatability becomes obvious. The problem of applying the theory of deterministic chaos as an early warning system for enterprises against bankruptcy undoubtedly requires further research. Chaos theory makes an important contribution to the modeling of investment processes. As far as the practical aspect for a stock market investor is concerned, it should be stated that it is possible to design the behavior of a joint-stock company on the Warsaw Stock Exchange in terms of its attractiveness in the short term, because there is a risk of sudden changes, when the limit after which forecasts disappear is exceeded, the interpretation of the results may then cause problems. A comprehensive analysis of all indicators resulting from chaos theory allows to formulate conclusions about the economic condition of a given joint-stock company. It should be stated that depending on the size of the ratio, it can be said whether a company is more or less attractive to recommend to a stock market investor in terms of its investment attractiveness on the Polish stock exchange. In the research, it was necessary to remember to take into account the specificity of a small stock exchange, which is the Warsaw Stock Exchange. Such a situation also requires individualisation of the interpretation of the results for individual joint-stock companies, because the assessment of the investment attractiveness of companies listed on small stock exchanges, such as the Warsaw Stock Exchange, on the basis of models

based on the theory of deterministic chaos, requires an individual approach to each of the entities listed on the stock exchange and to the economic situation and specificity of a given local financial market. It should also be remembered that fractal analysis does not bring us closer to forecasting the value of shares for a specific day, but it allows us to assess the probability of specific market behavior and model alternative scenarios of its behavior. Fractal analysis of the functioning of joint-stock companies on the Polish stock exchange is an important complement to classic methods of analysis. Despite many debatable aspects related to the use of deterministic chaos theory algorithms as a system of tuition warning against bankruptcy, their use is fully justified.

References

Altman E.I., 2000, *Predicting Financial Distress of Companies: Revisiting the Z-score and ZETA® Models*, International Insolvency Institute, New York.

Andrasik J., 2005, *Classification of mMethods for Forecasting the Risk of Bankruptcy of Enterprises*, "Barometr Regionalny", no. 1(4), pp. 15–22.

Aziz M.A., Dar H.A., 2004, *Predicting Corporate Bankruptcy: Whiter Do We Stand?*, [in:] E. Piątek, M. Konat-Staniek, *Methods of Identifying Insolvency and Assessment of the Risk of Bankruptcy*, "Finanse, Rynki Finansowe, Ubezpieczenia", no. 4/2017(88), pp. 375–382.

Bień W., 2011, *Corporate Finance Management*, Wydawnictwo Difin, Warszawa.

Bombiak E., 2010, *Discriminant Models as a Method of Assessing the Financial Situation of an Enterprise*, Scientific Papers of the Podlasie University in Siedlce, "Administration and Management", no. 86, pp. 141–152.

Bose I., Mohapatra R., 2001, *Business Data Mining – A Machine Learning Perspective*, "Information and Menagement Journal", no. 39, pp. 211–224.

Buła R., 2013, *Investment Risk and the Fractal Dimension*, "Studia Ekonomiczne", no. 155, pp. 450–466.

Buła R., 2017, *Analysis of the Fractal Dimension of Companies Listed on the Warsaw Stock Exchange – Methodological Aspects*, "Nauki o finansach", no. 1(30), pp. 9–27.

Buła R., 2019, *Implications of Fractal Market Theory for the Risk Assessment of Financial Investments*, KNF, Warszawa.

Dębkowska K., 2012, *Forecasting the Bankruptcy of Enterprises Using Selected methods of Multidimensional Statistical Analysis*, "Management and Finance", vol. 10, no. 1, pp. 175–184.

Gajdka J., Stos D., 1996, *The Use of Discriminant Analysis in the Assessment of the Financial Condition of Enterprises*, [in:] *Restructuring in the Process of Transformation and Development of Enterprises*, Borowiecki R. (ed.), Wydawnictwo Akademii Ekonomicznej w Krakowie, Kraków.

Gniadkowska-Szymańska A., Bolek M., 2018, *Assessment of the Economic Condition of an Enterprise on the Capital Market in the Light of Bankruptcy Law*, "Finanse, Rynki Finansowe, Ubezpieczenia", no. 1(91), pp.14–16.

Gołaszewski P., Urbanek P., Walińska E., 2001, *Analysis of Financial Statements*, Wydawnictwo FRR, Łódź.

Grochowina D., 2014, *The Influence of Data Imputation Methods on the Classification Efficiency of the Logit Model Used to Forecast the Bankruptcy of Enterprises*, "Ekonomia XLV", no. 2, pp. 188–189.

Hamrol M., Czajka B., Piechocki M., 2004, *Bankruptcy of the Enterprise – A Model of Discriminatory Analysis*, "Przegląd Organizacji, no. 6, pp. 35–39.

Jędrzejewski S., 2005, *Identification of the Risk of Bankruptcy of Enterprises*, Ostrów Wielkopolski.

Kisielińska J., Waszkowski A., 2010, *Polish Models for Forecasting the Bankruptcy of Enterprises and their Verification*, Scientific Papers of the Warsaw University of Life Sciences "Ekonomika i Organizacja Gospodarki Żywnościowej", no. 82, pp. 17–31.

Korol T., 2010, *Forecasting Bankruptcy of Companies Using Soft Calculation Techniques*, Financial Internet Quarterly, "e-Finanse" vol. 6, no. 2, pp. 1–15.

Letkowski D., 2014, *Verification of the Effectiveness of the Use of the Discriminant Function to Predict Changes in Share Prices*, "Finanse, Rynki Finansowe, Ubezpieczenia", no. 67, pp. 315–325.

Miosionek-Schweda M., 2014, *Altman's Model as a Tool for Assessing the Risk of Bankruptcy of Enterprises*, "Prace Naukowe Uniwersytetu Ekonomicznygo we Wrocławiu", no. 342, pp. 164–175.

Orzeszko W., 2005, *Identification and Forecasting of Deterministic Chaos in Economic Time Series*, Polskie Towarzystwo Ekonomiczne, Warszawa.

Peters E.E., 1997, *Chaos Theory and Capital Markets*, Wydawnictwo Wig Press, Warszawa.

Ptak-Chmielewska A., 2012, *The Use of Survival Models and Discriminant Analysis to Assess the Risk of Bankruptcy of Enterprises*, "Ekonometria", no. 4(38), pp. 157–172.

Siemieniuk N., 2001, *Fractals of the Properties of the Polish Capital Market*, Wydawnictwo Uniwersytetu w Białymstoku, Białystok.

Sierpińska M., Jachna T., 1997, *Evaluation of the Enterprise According to World Standards*, Wydawnictwo Naukowe PWN, Warszawa.

Stabryła A., 2002, *Strategic Management in the Theory and Practice of the Company*, Wydawnictwo Naukowe PWN, Warszawa–Kraków.

Statistical Methods of Forecasting Bankruptcy in the Changing Economic Situation, 2014, Pociecha J. (ed.), Wydawnictwo Fundacja Uniwersytetu Ekonomicznego w Krakowie, Kraków.

Wajda P., 2009, *The Use of Discriminant Analysis in the Assessment of the Risk of Bankruptcy of Entrepreneurs*, Przegląd Corporate Governance, vol. 1, pp. 29–34.

Waśniewski T., Skoczylas W., 2004, *Theory and Practice of Financial Analysis in the Enterprise*, Wydawnictwo FRRwP, Warszawa.

Weitzel W., Johnson E., 1999, *Decline In Organizations – A Literature Integration and Extension*, [in:] L. Bednarski, *Symptoms and Assessment of Threats to the Financial Situation of the Enterprise*, "Money and Bond", no. 1(2), pp. 44–46.

Wojnar J., *Evaluation of the Effectiveness of Discriminant Analysis Models for Forecasting the Financial Risk of Listed Companies*, http://zn.mwse.edu.pl/ebooki/24/219-231.pdf [date of access: on: 25.08.2024].

www 1, https://zn.mwse.edu.pl/index.php/zn/issue/view/24/zn24c [date of access: 25.08.2024].

www 2, http://www.iwiedza.net/download/chaos.pdf [date of access: 12.09.2024]