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Elementary Number Theory Problems. Part XII – Primes in Arithmetic Progression

Adam Grabowski
Faculty of Computer Science
University of Białystok
Poland

Summary. In this paper another twelve problems from W. Sierpiński's book "250 Problems in Elementary Number Theory" are formalized, using the Mizar formalism, namely: 42, 43, 51, 51a, 57, 59, 72, 135, 136, and 153–155. Significant amount of the work is devoted to arithmetic progressions.

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Introduction

This article contains solutions of selected problems from W. Sierpiński's book "250 Problems in Elementary Number Theory" [12] – the work outlined in [8]. We make an extensive use of the general notion of arithmetic progression developed previously in [2] and results on prime and composite numbers [7].

The preliminary part of the article contains the proof of Theorem 5 from [11], p. 121 (credited to Cantor) stating that if n and r are natural numbers, $n > 1$ and if n terms of the arithmetical progression $m, m+r, \ldots, m+(n-1)r$ are odd prime numbers, then the difference r is divisible by every prime less than n (see [1], vol. I, p. 425). It is used to solve Problem 72, that an increasing arithmetic progression with ten terms, formed of primes, with the least possible last term is the one with the first term 199 and difference 210.

Problems 42, 43, 51, and 51a are taken from Section II ("Relatively prime numbers"), Problems 57, 59, and 72 are from Section III ("Arithmetic progressions"), the rest, i.e. Problems 135, 136 – from Section IV ("Prime and composite numbers").

Problem 42 is closely connected to polygonal numbers formalized in [3].

Problems 153–155, taken from Section V ("Diophantine equations") deal with the solution of the equation

$$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = k$$

in positive integers x, y, and z, where k is equal to one, two, and three, respectively. More general idea of the problem (open in [12]), about positive integer solution of this equation with arbitrary natural k is discussed quite recently in [13].

Proofs of other problems are straightforward formalizations of solutions given in the book, by means of available development of number theory in Mizar [4], [5], using ellipsis [6] extensively, looking forward for more advanced automatization of arithmetical calculations [9].

1. Preliminaries

Now we state the proposition:

(1) Let us consider objects x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}. Then $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} = \{x_1, x_2, x_3, x_4, x_5\} \cup \{x_6, x_7, x_8, x_9, x_{10}\}$.

Let m be a composite natural number and n be a non zero natural number. Let us observe that $m \cdot n$ is composite. Let m, n be non zero, non trivial natural numbers. Observe that $m \cdot n$ is composite. Let r be a real number. Let us observe that r^2 is non negative.

Let k be a natural number and n be a non zero, non trivial natural number. Let us observe that $k + n$ is non trivial and non zero and $k + 1$ is non zero and $k + 2$ is non trivial and non zero and $k + 3$ is non trivial and non zero. Now we state the propositions:

(2) Let us consider a natural number n. Suppose $n \bmod 11 = 1$ and $n \bmod 2 = 1$. Then $n \bmod 22 = 1$.

(3) Let us consider natural numbers m, n, r. Suppose $n > 1$ and for every natural number i such that $0 \leqslant i < n$ holds $(\mathrm{ArProg}(m, r))(i)$ is odd and prime. Let us consider a prime number p. If $p < n$, then $p \mid r$.

2. Problem 42

Now we state the proposition:

(4) Let us consider natural numbers a, m, n. If a and m are relatively prime and $n \mid a$, then n and m are relatively prime.

Let us consider a natural number a. Now we state the propositions:

(5) a and $2 \cdot a + 1$ are relatively prime.

(6) a and $6 \cdot a + 1$ are relatively prime.

(7) a and $3 \cdot a + 1$ are relatively prime.

(8) Let us consider an increasing finite sequence f of elements of $\mathbb{N}$, and a natural number x. Suppose for every natural number i such that $i \in \operatorname{dom} f$ holds $f(i) < x$. Then $f \frown \langle x \rangle$ is increasing.

Proof: Consider k being a natural number such that $\operatorname{dom} f = \operatorname{Seg} k$. Set $f_4 = f \frown \langle x \rangle$. For every natural numbers m, n such that $m, n \in \operatorname{dom} f_4$ and $m < n$ holds $f_4(m) < f_4(n)$. $\square$

Let us consider a natural number n. Now we state the propositions:

(9) $\operatorname{Seg} 1 \longmapsto n$ is an increasing finite sequence of elements of $\mathbb{N}$.

Proof: Set $f = \operatorname{Seg} 1 \longmapsto n$. For every natural numbers m, n such that $m, n \in \operatorname{dom} f$ and $m < n$ holds $f(m) < f(n)$. $\square$

(10) There exists an increasing, non-empty finite sequence f of elements of $\mathbb{N}$ such that

(i) $\operatorname{dom} f = \operatorname{Seg}(n + 1)$, and

(ii) for every natural number i such that $i \in \operatorname{dom} f$ holds $f(i)$ is triangular, and

(iii) f is with all coprime terms.

Proof: Define $\mathcal{P}[\text{natural number}] \equiv$ there exists an increasing, non-empty finite sequence f of elements of $\mathbb{N}$ such that $\operatorname{dom} f = \operatorname{Seg}(\$_1 + 1)$ and for every natural number i such that $i \in \operatorname{dom} f$ holds $f(i)$ is triangular and f is with all coprime terms. $\mathcal{P}[0]$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k + 1]$. For every natural number n, $\mathcal{P}[n]$.

Consider f being an increasing, non-empty finite sequence of elements of $\mathbb{N}$ such that $\operatorname{dom} f = \operatorname{Seg}(n + 1)$ and for every natural number i such that $i \in \operatorname{dom} f$ holds $f(i)$ is triangular and f is with all coprime terms. $\square$

3. Problem 43

Let n be a natural number. The functor Tetrahedron(n) yielding a natural number is defined by the term

(Def. 1) $\frac{n\cdot(n+1)\cdot(n+2)}{6}$.

We say that n is tetrahedral if and only if

(Def. 2) there exists a natural number k such that $n = \mathrm{Tetrahedron}(k)$.

Now we state the proposition:

(11) Let us consider a natural number n. Then there exists an increasing, non-empty finite sequence f of elements of $\mathbb{N}$ such that

(i) $\mathrm{dom}\, f = \mathrm{Seg}(n+1)$, and

(ii) for every natural number i such that $i \in \mathrm{dom}\, f$ holds $f(i)$ is tetrahedral, and

(iii) f is with all coprime terms.

Proof: Define $\mathcal{P}$[natural number] $\equiv$ there exists an increasing, non-empty finite sequence f of elements of $\mathbb{N}$ such that $\mathrm{dom}\, f = \mathrm{Seg}(\$_1 + 1)$ and for every natural number i such that $i \in \mathrm{dom}\, f$ holds $f(i)$ is tetrahedral and f is with all coprime terms. $\mathcal{P}[0]$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. For every natural number n, $\mathcal{P}[n]$.

Consider f being an increasing, non-empty finite sequence of elements of $\mathbb{N}$ such that $\mathrm{dom}\, f = \mathrm{Seg}(n+1)$ and for every natural number i such that $i \in \mathrm{dom}\, f$ holds $f(i)$ is tetrahedral and f is with all coprime terms. $\square$

4. Problem 51

Let us consider a non zero natural number n. Now we state the propositions:

(12) $\gcd(n, \mathrm{Fermat}\, n) = 1$.

(13) n and $\mathrm{Fermat}\, n$ are relatively prime.

5. Problem 51a

Now we state the propositions:

(14) Let us consider natural numbers n, k, m. Suppose $n \mid k \cdot m$. Then there exist natural numbers a, b such that

(i) $a \mid k$, and

(ii) $b \mid m$, and

(iii) $n = a \cdot b$.

(15) Let us consider a set A. Suppose $A = \{n$, where n is a non zero natural number : $\gcd(n, 2^n - 1) > 1\}$. Then

(i) A is infinite, and

(ii) for every natural number k such that $k \in A$ holds $k \geqslant 6$.

PROOF: For every non zero natural number k, $\gcd(6 \cdot k, 2^{6 \cdot k} - 1) \geqslant 3$. For every non zero natural number k, $6 \cdot k \in A$. For every natural number m, there exists a natural number n such that $n \geqslant m$ and $n \in A$. For every natural number k such that $k \in A$ holds $k \geqslant 6$ by [14, (5)]. $\square$

6. PROBLEM 57

Let us consider positive natural numbers a, b and natural numbers k, x, m. Now we state the propositions:

(16) If $(\mathrm{ArProg}(b, a))(k) = x^{\mathbf{2}}$, then $(\mathrm{ArProg}(b, a))(m^{\mathbf{2}} \cdot a + 2 \cdot m \cdot x + k) = (m \cdot a + x)^{\mathbf{2}}$.

(17) If $(\mathrm{ArProg}(b, a))(k) = x^{\mathbf{2}}$, then $(\mathrm{ArProg}(b, a))(m^{\mathbf{2}} \cdot a + 2 \cdot m \cdot x + k)$ is a square.

(18) Let us consider non zero natural numbers m, n. Suppose m is quadratic residue modulo n. Then there exists a natural number i such that $(\mathrm{ArProg}(m, n))(i)$ is a square.

(19) Let us consider non zero natural numbers m, n, and a set A. Suppose $A = \{i$, where i is a natural number : $(\mathrm{ArProg}(m, n))(i)$ is a square$\}$. Then A is infinite if and only if m is quadratic residue modulo n.

PROOF: Consider i being a natural number such that $(\mathrm{ArProg}(m, n))(i)$ is a square. Consider x being a natural number such that $(\mathrm{ArProg}(m, n))(i) = x^{\mathbf{2}}$. For every natural number j, there exists a natural number k such that $k \geqslant j$ and $k \in A$. $\square$

7. PROBLEM 59

Now we state the proposition:

(20) Let us consider a natural number k. If $k > 1$, then $k \cdot k \nmid k$.

Observe that there exists an arithmetic progression which is non-empty, natural-valued, and increasing. Now we state the propositions:

(21) Let us consider a natural number n, and a prime number p. If n is perfect power and $p \mid n$, then $p^2 \mid n$.

(22) There exists no non-empty, natural-valued, increasing arithmetic progression f such that for every natural numbers i, $\mathcal{N}$ such that $\mathcal{N} = f(i)$ holds $\mathcal{N}$ is perfect power.

Proof: Consider f being a non-empty, natural-valued, increasing arithmetic progression such that for every natural numbers i, $\mathcal{N}$ such that $\mathcal{N} = f(i)$ holds $\mathcal{N}$ is perfect power. Reconsider $b = f(0)$ as a natural number. Reconsider $a = \text{difference}(f)$ as a natural number.

Consider p being a prime number such that $p > a + b$. Reconsider $p_2 = p^{\mathbf{2}}$ as a natural number. $\gcd(a, p) = 1$. Consider x, y being natural numbers such that $a \cdot x - p_2 \cdot y = 1$. Reconsider $k = (p - b) \cdot x$ as a natural number. Reconsider $a_1 = a \cdot k + b$ as a natural number. $p_2 \nmid a \cdot k + b$. a_1 is not perfect power. □

8. Problem 72

Now we state the propositions:

(23) Let us consider an arithmetic progression f. Suppose for every natural number i, $f(i)$ is a prime number. Then difference(f) is an integer.

(24) Let us consider prime numbers p, q. If $p - q$ is odd, then $p = 2$ or $q = 2$.

Let p, q be prime numbers. One can check that $p-q$ is integer. Let p, q be greater than 2 prime numbers. Observe that $p - q$ is even. Let us consider an increasing arithmetic progression f. Now we state the propositions:

(25) If for every natural number i, $f(i)$ is a prime number, then $f(1) > 2$.

(26) If for every natural number i, $f(i)$ is a prime number, then difference(f) is an even natural number. The theorem is a consequence of (25).

(27) $(\text{ArProg}(199, 210))(0) = 199$.

(28) $(\text{ArProg}(199, 210))(1) = 409$. The theorem is a consequence of (27).

(29) $(\text{ArProg}(199, 210))(2) = 619$. The theorem is a consequence of (28).

(30) $(\text{ArProg}(199, 210))(3) = 829$. The theorem is a consequence of (29).

(31) $(\text{ArProg}(199, 210))(4) = 1039$. The theorem is a consequence of (30).

(32) $(\text{ArProg}(199, 210))(5) = 1249$. The theorem is a consequence of (31).

(33) $(\text{ArProg}(199, 210))(6) = 1459$. The theorem is a consequence of (32).

(34) $(\text{ArProg}(199, 210))(7) = 1669$. The theorem is a consequence of (33).

(35) $(\text{ArProg}(199, 210))(8) = 1879$. The theorem is a consequence of (34).

(36) $(\text{ArProg}(199, 210))(9) = 2089$. The theorem is a consequence of (35).

Let f be a natural-valued arithmetic progression. One can verify that difference(f) is integer. Let us consider an increasing, natural-valued arithmetic progression f. Now we state the propositions:

(37) If for every natural number i such that $0 \leqslant i < 10$ holds $f(i)$ is an odd prime number, then $210 \mid \mathrm{difference}(f)$. The theorem is a consequence of (3).

(38) If for every natural number i such that $0 \leqslant i < 10$ holds $f(i)$ is an odd prime number, then $\mathrm{difference}(f) \geqslant 210$.

(39) Let us consider an increasing, natural-valued arithmetic progression f. Suppose for every natural number i such that $0 \leqslant i < 10$ holds $f(i)$ is an odd prime number and $\mathrm{difference}(f) = 210$. Let us consider a natural number f_0. If $f_0 = f(0)$, then $f_0 \bmod 11 = 1$.

PROOF: $f_0 \bmod 11 \neq 0$. $f_0 \bmod 11 \neq 10$. $f_0 \bmod 11 \neq 9$. $f_0 \bmod 11 \neq 8$. $f_0 \bmod 11 \neq 7$. $f_0 \bmod 11 \neq 6$. $f_0 \bmod 11 \neq 5$. $f_0 \bmod 11 \neq 4$. $f_0 \bmod 11 \neq 3$. $f_0 \bmod 11 \neq 2$. □

Let us consider an increasing, natural-valued arithmetic progression f. Now we state the propositions:

(40) If for every natural number i such that $0 \leqslant i < 10$ holds $f(i)$ is an odd prime number and $\mathrm{difference}(f) = 210$, then $f(0) \geqslant 199$.

PROOF: $f(0) \bmod 11 = 1$. $f(0) \bmod 22 = 1$. If $f(0) \operatorname{div} 22 = 0$, then $f(0) = 1$. If $f(0) \operatorname{div} 22 = 1$, then $f(0) = 23$. If $f(0) \operatorname{div} 22 = 2$, then $f(0) = 45$. If $f(0) \operatorname{div} 22 = 3$, then $f(0) = 67$. If $f(0) \operatorname{div} 22 = 4$, then $f(0) = 89$. If $f(0) \operatorname{div} 22 = 5$, then $f(0) = 111$. If $f(0) \operatorname{div} 22 = 6$, then $f(0) = 133$. If $f(0) \operatorname{div} 22 = 7$, then $f(0) = 155$. If $f(0) \operatorname{div} 22 = 8$, then $f(0) = 177$. If $f(0) \operatorname{div} 22 > 4$, then $f(0) \geqslant 199$. $f(0) \neq 23$. $f(0) \neq 67$. $f(0) \neq 89$. □

(41) If for every natural number i such that $0 \leqslant i < 10$ holds $f(i)$ is an odd prime number, then $f(9) \geqslant 2089$. The theorem is a consequence of (37), (40), and (38).

(42) $\mathrm{rng}(\mathrm{ArProg}(199, 210) \restriction 10) = \{199, 409, 619, 829, 1039, 1249, 1459, 1669, 1879, 2089\}$.

PROOF: Set $g = \mathrm{ArProg}(199, 210)$. $\mathrm{rng}(\mathrm{ArProg}(199, 210) \restriction 10) \subseteq \{199, 409, 619, 829, 1039, 1249, 1459, 1669, 1879, 2089\}$. $x = g(0)$ or $x = g(1)$ or $x = g(2)$ or $x = g(3)$ or $x = g(4)$ or $x = g(5)$ or $x = g(6)$ or $x = g(7)$ or $x = g(8)$ or $x = g(9)$. $x \in \mathrm{rng}(\mathrm{ArProg}(199, 210) \restriction 10)$. □

(43) $\overline{\overline{\mathrm{rng}(\mathrm{ArProg}(199, 210) \restriction 10) \cap \mathbb{P}}} = 10$.

PROOF: Set $f = \mathrm{ArProg}(199, 210) \restriction 10$. $\{199, 409, 619, 829, 1039, 1249, 1459, 1669, 1879, 2089\} \subseteq \mathrm{rng}\, f \cap \mathbb{P}$. $\{199, 409, 619, 829, 1039\}$ misses $\{1249, 1459, 1669, 1879, 2089\}$. $\mathrm{rng}\, f \cap \mathbb{P} \subseteq \{199, 409, 619, 829, 1039, 1249, 1459, 1669, 1879, 2089\}$. □

9. Problem 135

Now we state the proposition:

(44) Let us consider a prime number p. Suppose $p+2$ is a prime number and $p+6$ is a prime number and $p+8$ is a prime number and $p+12$ is a prime number and $p+14$ is a prime number. Then $p=5$.

10. Problem 136

Let n be an integer. The functor PrimeDivisors(n) yielding a subset of $\mathbb{N}$ is defined by the term

(Def. 3) $\{k$, where k is a prime number : $k \mid n\}$.

Now we state the propositions:

(45) Let us consider an integer i. Then PrimeDivisors(i) $\subseteq \mathbb{P}$.

(46) Let us consider a non zero natural number n. Then PrimeDivisors(n) $\subseteq$ $\mathrm{Seg}\, n$.

(47) Let us consider a natural number n. Then PrimeDivisors(n) $\subseteq$ the set of positive divisors of n.

(48) Let us consider natural numbers a, b. Then PrimeDivisors($a \cdot b$) $=$ PrimeDivisors(a) $\cup$ PrimeDivisors(b).
Proof: PrimeDivisors($a \cdot b$) $\subseteq$ PrimeDivisors(a)$\cup$PrimeDivisors(b) by [10, (7)]. $\square$

(49) Let us consider a natural number n, and a natural number a. If $n \geqslant 1$, then PrimeDivisors(a^n) $=$ PrimeDivisors(a).
Proof: PrimeDivisors(a^n) $\subseteq$ PrimeDivisors(a). Consider k being a prime number such that $k = x$ and $k \mid a$. $\square$

(50) Let us consider a natural number k, and a prime number p. If $k \geqslant 1$, then PrimeDivisors(p^k) $= \{p\}$.
Proof: Define $\mathcal{P}$[natural number] $\equiv$ PrimeDivisors($p^{\$_1+1}$) $= \{p\}$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. For every natural number n, $\mathcal{P}[n]$. $\square$

(51) PrimeDivisors(1) $= \emptyset$.

Let us consider a natural number k. Now we state the propositions:

(52) If $k \geqslant 1$, then PrimeDivisors($2^k \cdot (2^k - 2)$) $= \{2\} \cup$ PrimeDivisors($2^{k-'1} - 1$). The theorem is a consequence of (48) and (50).

(53) If $k \geqslant 1$, then PrimeDivisors($2^k - 2$) $= \{2\} \cup$ PrimeDivisors($2^{k-'1} - 1$). The theorem is a consequence of (48).

(54) PrimeDivisors$(2^k \cdot (2^k - 2) + 1)$ = PrimeDivisors$(2^k - 1)$. The theorem is a consequence of (48).

(55) Let us consider a natural number a. Then PrimeDivisors$(a \cdot a)$ = PrimeDivisors(a). The theorem is a consequence of (48).

(56) Let us consider natural numbers k, m, n. Suppose $k \geqslant 1$ and $m = 2^k - 2$ and $n = 2^k \cdot (2^k - 2)$. Then

(i) PrimeDivisors(m) = PrimeDivisors(n), and

(ii) PrimeDivisors$(m + 1)$ = PrimeDivisors$(n + 1)$.

The theorem is a consequence of (54), (53), and (52).

(57) (i) PrimeDivisors(75) = PrimeDivisors(1215), and

(ii) PrimeDivisors$(75 + 1)$ = PrimeDivisors$(1215 + 1)$.

The theorem is a consequence of (48) and (55).

11. PROBLEM 153

Now we state the propositions:

(58) Let us consider positive real numbers x, y, z. Then $\frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{x} = 1$.

(59) There exist no positive natural numbers x, y, z such that $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 1$. The theorem is a consequence of (58).

12. PROBLEM 154

Now we state the propositions:

(60) Let us consider a positive real number a, and a positive natural number n. Then $\sqrt[n]{a}$ is positive.

(61) Let us consider positive real numbers a, b, c. If it is not true that $a = b$ and $b = c$, then $(\frac{a+b+c}{3})^3 > a \cdot b \cdot c$. The theorem is a consequence of (60).

(62) There exist no positive natural numbers x, y, z such that $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 2$. The theorem is a consequence of (58) and (61).

13. PROBLEM 155

Now we state the proposition:

(63) Let us consider positive natural numbers x, y, z. If $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 3$, then $x = y$ and $y = z$. The theorem is a consequence of (61) and (58).

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