FORMALIZED MATHEMATICS
Vol. 28, No. 4, Pages 269–278, 2020
DOI: 10.2478/forma-2020-0024

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General Theory and Tools for Proving Algorithms in Nominative Data Systems

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Summary. In this paper we introduce some new definitions for sequences of operations and extract general theorems about properties of iterative algorithms encoded in nominative data language [20] in the Mizar system [3], [1] in order to simplify the process of proving algorithms in the future.

This paper continues verification of algorithms [10], [13], [12], [14] written in terms of simple-named complex-valued nominative data [6], [8], [18], [11], [15], [16].

The validity of the algorithm is presented in terms of semantic Floyd-Hoare triples over such data [9]. Proofs of the correctness are based on an inference system for an extended Floyd-Hoare logic [2], [4] with partial pre- and post-conditions [17], [19], [7], [5].

MSC: 68Q60 03B70 68V20

Keywords: nominative data; program verification; inference rules

MML identifier: NOMIN_8, version: 8.1.10 5.64.1388

1. Composition Rules for Programs

Let D be a non empty set. One can verify that there exists a finite sequence which is non empty and D-valued.

Let n be a natural number. One can verify that there exists a finite sequence which is D-valued and n-element.

From now on D denotes a non empty set, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10} denote binominative functions of D, p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}

ISSN 1426–2630(Print), 1898-9934(Online)

denote partial predicates of D, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10} denote total partial predicates of D, n, m, N denote natural numbers, f_D denotes a $(D \dot{\to} D)$-valued finite sequence, f_B denotes a $(D \dot{\to} Boolean)$-valued finite sequence, V, A denote sets.

From now on *val* denotes a function, *loc* denotes a V-valued function, d_1 denotes a non-atomic nominative data of V and A, p denotes a partial predicate over simple-named complex-valued nominative data of V and A, d, v denote objects, z_2 denotes a non zero natural number, *inp*, *pos* denote finite sequences, and *prg* denotes a non empty, $(\text{FPrg}(\text{ND}_{\text{SC}}(V, A)))$-valued finite sequence.

Let us consider D, f_1, f_2, f_3, f_4, f_5, f_6, and f_7. The functor PP-composition $(f_1, f_2, f_3, f_4, f_5, f_6, f_7)$ yielding a binominative function of D is defined by the term

(Def. 1) PP-composition$(f_1, f_2, f_3, f_4, f_5, f_6) \bullet f_7$.

Now we state the proposition:

(1) Unconditional composition rule for 7 programs:
Suppose $\langle p_1, f_1, p_2 \rangle$ is an SFHT of D and $\langle p_2, f_2, p_3 \rangle$ is an SFHT of D and $\langle p_3, f_3, p_4 \rangle$ is an SFHT of D and $\langle p_4, f_4, p_5 \rangle$ is an SFHT of D and $\langle p_5, f_5, p_6 \rangle$ is an SFHT of D and $\langle p_6, f_6, p_7 \rangle$ is an SFHT of D and $\langle p_7, f_7, p_8 \rangle$ is an SFHT of D and $\langle \sim p_2, f_2, p_3 \rangle$ is an SFHT of D and $\langle \sim p_3, f_3, p_4 \rangle$ is an SFHT of D and $\langle \sim p_4, f_4, p_5 \rangle$ is an SFHT of D and $\langle \sim p_5, f_5, p_6 \rangle$ is an SFHT of D and $\langle \sim p_6, f_6, p_7 \rangle$ is an SFHT of D and $\langle \sim p_7, f_7, p_8 \rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6, f_7), p_8 \rangle$ is an SFHT of D.

Let us consider D, f_1, f_2, f_3, f_4, f_5, f_6, f_7, and f_8. The functor PP-composition$(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8)$ yielding a binominative function of D is defined by the term

(Def. 2) PP-composition$(f_1, f_2, f_3, f_4, f_5, f_6, f_7) \bullet f_8$.

Now we state the proposition:

(2) Unconditional composition rule for 8 programs:
Suppose $\langle p_1, f_1, p_2 \rangle$ is an SFHT of D and $\langle p_2, f_2, p_3 \rangle$ is an SFHT of D and $\langle p_3, f_3, p_4 \rangle$ is an SFHT of D and $\langle p_4, f_4, p_5 \rangle$ is an SFHT of D and $\langle p_5, f_5, p_6 \rangle$ is an SFHT of D and $\langle p_6, f_6, p_7 \rangle$ is an SFHT of D and $\langle p_7, f_7, p_8 \rangle$ is an SFHT of D and $\langle p_8, f_8, p_9 \rangle$ is an SFHT of D and $\langle \sim p_2, f_2, p_3 \rangle$ is an SFHT of D and $\langle \sim p_3, f_3, p_4 \rangle$ is an SFHT of D and $\langle \sim p_4, f_4, p_5 \rangle$ is an SFHT of D and $\langle \sim p_5, f_5, p_6 \rangle$ is an SFHT of D and $\langle \sim p_6, f_6, p_7 \rangle$ is an SFHT of D and $\langle \sim p_7, f_7, p_8 \rangle$ is an SFHT of D and $\langle \sim p_8, f_8, p_9 \rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8), p_9 \rangle$ is an SFHT of D. The theorem is a consequence of (1).

Let us consider D, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, and f_9. The functor PP-composition($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9$) yielding a binominative function of D is defined by the term

(Def. 3) PP-composition($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8$) $\bullet f_9$.

Now we state the proposition:

(3) Unconditional composition rule for 9 programs:
Suppose $\langle p_1, f_1, p_2\rangle$ is an SFHT of D and $\langle p_2, f_2, p_3\rangle$ is an SFHT of D and $\langle p_3, f_3, p_4\rangle$ is an SFHT of D and $\langle p_4, f_4, p_5\rangle$ is an SFHT of D and $\langle p_5, f_5, p_6\rangle$ is an SFHT of D and $\langle p_6, f_6, p_7\rangle$ is an SFHT of D and $\langle p_7, f_7, p_8\rangle$ is an SFHT of D and $\langle p_8, f_8, p_9\rangle$ is an SFHT of D and $\langle p_9, f_9, p_{10}\rangle$ is an SFHT of D and $\langle \sim p_2, f_2, p_3\rangle$ is an SFHT of D and $\langle \sim p_3, f_3, p_4\rangle$ is an SFHT of D and $\langle \sim p_4, f_4, p_5\rangle$ is an SFHT of D and $\langle \sim p_5, f_5, p_6\rangle$ is an SFHT of D and $\langle \sim p_6, f_6, p_7\rangle$ is an SFHT of D and $\langle \sim p_7, f_7, p_8\rangle$ is an SFHT of D and $\langle \sim p_8, f_8, p_9\rangle$ is an SFHT of D and $\langle \sim p_9, f_9, p_{10}\rangle$ is an SFHT of D. Then $\langle p_1$, PP-composition($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9$), $p_{10}\rangle$ is an SFHT of D. The theorem is a consequence of (2).

Let us consider D, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, and f_{10}. The functor PP-composition($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}$) yielding a binominative function of D is defined by the term

(Def. 4) PP-composition($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9$) $\bullet f_{10}$.

Now we state the propositions:

(4) Unconditional composition rule for 10 programs:
Suppose $\langle p_1, f_1, p_2\rangle$ is an SFHT of D and $\langle p_2, f_2, p_3\rangle$ is an SFHT of D and $\langle p_3, f_3, p_4\rangle$ is an SFHT of D and $\langle p_4, f_4, p_5\rangle$ is an SFHT of D and $\langle p_5, f_5, p_6\rangle$ is an SFHT of D and $\langle p_6, f_6, p_7\rangle$ is an SFHT of D and $\langle p_7, f_7, p_8\rangle$ is an SFHT of D and $\langle p_8, f_8, p_9\rangle$ is an SFHT of D and $\langle p_9, f_9, p_{10}\rangle$ is an SFHT of D and $\langle p_{10}, f_{10}, p_{11}\rangle$ is an SFHT of D and $\langle \sim p_2, f_2, p_3\rangle$ is an SFHT of D and $\langle \sim p_3, f_3, p_4\rangle$ is an SFHT of D and $\langle \sim p_4, f_4, p_5\rangle$ is an SFHT of D and $\langle \sim p_5, f_5, p_6\rangle$ is an SFHT of D and $\langle \sim p_6, f_6, p_7\rangle$ is an SFHT of D and $\langle \sim p_7, f_7, p_8\rangle$ is an SFHT of D and $\langle \sim p_8, f_8, p_9\rangle$ is an SFHT of D and $\langle \sim p_9, f_9, p_{10}\rangle$ is an SFHT of D and $\langle \sim p_{10}, f_{10}, p_{11}\rangle$ is an SFHT of D. Then $\langle p_1$, PP-composition($f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}$), $p_{11}\rangle$ is an SFHT of D. The theorem is a consequence of (3).

(5) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, p_2\rangle$ is an SFHT of D. Then $\langle p_1, f_1 \bullet f_2, p_2\rangle$ is an SFHT of D.

(6) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, p_2\rangle$ is an SFHT of D. Then $\langle p_1$, PP-composition(f_1, f_2, f_3), $p_2\rangle$ is an SFHT of D. The theorem is a consequence of (5).

(7) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, p_2\rangle$ is an SFHT of D. Then

$\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (6).

(8) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, q_4\rangle$ is an SFHT of D and $\langle q_4, f_5, p_2\rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (7).

(9) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, q_4\rangle$ is an SFHT of D and $\langle q_4, f_5, q_5\rangle$ is an SFHT of D and $\langle q_5, f_6, p_2\rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (8).

(10) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, q_4\rangle$ is an SFHT of D and $\langle q_4, f_5, q_5\rangle$ is an SFHT of D and $\langle q_5, f_6, q_6\rangle$ is an SFHT of D and $\langle q_6, f_7, p_2\rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6, f_7), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (9).

(11) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, q_4\rangle$ is an SFHT of D and $\langle q_4, f_5, q_5\rangle$ is an SFHT of D and $\langle q_5, f_6, q_6\rangle$ is an SFHT of D and $\langle q_6, f_7, q_7\rangle$ is an SFHT of D and $\langle q_7, f_8, p_2\rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (10).

(12) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, q_4\rangle$ is an SFHT of D and $\langle q_4, f_5, q_5\rangle$ is an SFHT of D and $\langle q_5, f_6, q_6\rangle$ is an SFHT of D and $\langle q_6, f_7, q_7\rangle$ is an SFHT of D and $\langle q_7, f_8, q_8\rangle$ is an SFHT of D and $\langle q_8, f_9, p_2\rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (11).

(13) Suppose $\langle p_1, f_1, q_1\rangle$ is an SFHT of D and $\langle q_1, f_2, q_2\rangle$ is an SFHT of D and $\langle q_2, f_3, q_3\rangle$ is an SFHT of D and $\langle q_3, f_4, q_4\rangle$ is an SFHT of D and $\langle q_4, f_5, q_5\rangle$ is an SFHT of D and $\langle q_5, f_6, q_6\rangle$ is an SFHT of D and $\langle q_6, f_7, q_7\rangle$ is an SFHT of D and $\langle q_7, f_8, q_8\rangle$ is an SFHT of D and $\langle q_8, f_9, q_9\rangle$ is an SFHT of D and $\langle q_9, f_{10}, p_2\rangle$ is an SFHT of D. Then $\langle p_1, \text{PP-composition}(f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}), p_2\rangle$ is an SFHT of D. The theorem is a consequence of (12).

Let us consider D and f_D. Assume $0 < \text{len}\, f_D$. The functor PP-composition-Seq(f_D) yielding a finite sequence of elements of $D \dot{\rightarrow} D$ is defined by

(Def. 5) $\text{len}\, it = \text{len}\, f_D$ and $it(1) = f_D(1)$ and for every natural number n such that $1 \leqslant n < \text{len}\, f_D$ holds $it(n+1) = it(n) \bullet f_D(n+1)$.

The functor PP-composition(f_D) yielding a binominative function of D is defined by the term

(Def. 6) (PP-compositionSeq(f_D))(len PP-compositionSeq(f_D)).

Let us consider f_B. We say that f_D and f_B are composable if and only if

(Def. 7) $1 \leqslant \operatorname{len} f_D$ and $\operatorname{len} f_B = \operatorname{len} f_D + 1$ and for every n such that $1 \leqslant n \leqslant \operatorname{len} f_D$ holds $\langle f_B(n), f_D(n), f_B(n+1)\rangle$ is an SFHT of D and for every n such that $2 \leqslant n \leqslant \operatorname{len} f_D$ holds $\langle \sim f_B(n), f_D(n), f_B(n+1)\rangle$ is an SFHT of D.

Now we state the proposition:

(14) Composition rule for sequences of programs:
Suppose f_D and f_B are composable. Then $\langle f_B(1), \text{PP-composition}(f_D), f_B(\operatorname{len} f_B)\rangle$ is an SFHT of D.
Proof: Set $G = \text{PP-compositionSeq}(f_D)$. Define $\mathcal{P}[\text{natural number}] \equiv$ if $1 \leqslant \$_1 \leqslant \operatorname{len} f_D$, then $\langle f_B(1), G(\$_1), f_B(\$_1 + 1)\rangle$ is an SFHT of D. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. For every natural number k, $\mathcal{P}[k]$. $\square$

2. Values and Locations Validation

Let us consider V and A. Let val be a finite sequence. The functor $\Rightarrow(V, A, val)$ yielding a finite sequence of elements of $\mathrm{ND}_{\mathrm{SC}}(V, A) \dot{\to} \mathrm{ND}_{\mathrm{SC}}(V, A)$ is defined by

(Def. 8) $\operatorname{len} it = \operatorname{len} val$ and for every natural number n such that $1 \leqslant n \leqslant \operatorname{len} it$ holds $it(n) = val(n) \Rightarrow_a$.

Let us consider loc. Assume $\operatorname{len} val > 0$. Let p be a partial predicate over simple-named complex-valued nominative data of V and A. The functor ScPsuperposSeq(loc, val, p) yielding a finite sequence of elements of $\mathrm{ND}_{\mathrm{SC}}(V, A) \dot{\to} Boolean$ is defined by

(Def. 9) $\operatorname{len} it = \operatorname{len} val$ and $it(1) = \mathrm{S_P}(p, val(\operatorname{len} val) \Rightarrow_a, loc_{/\operatorname{len} val})$ and for every natural number n such that $1 \leqslant n < \operatorname{len} it$ holds $it(n+1) = \mathrm{S_P}(it(n), val(\operatorname{len} val - n) \Rightarrow_a, loc_{/\operatorname{len} val - n})$.

Now we state the proposition:

(15) Let us consider a non zero natural number z_2, and a z_2-element finite sequence val. Suppose loc, val and z_2 are correct w.r.t. d_1 and $1 \leqslant n \leqslant \operatorname{len} \text{LocalOverlapSeq}(A, loc, val, d_1, z_2)$ and $1 \leqslant m \leqslant \operatorname{len} \text{LocalOverlapSeq}(A, loc, val, d_1, z_2)$. Then $(\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(n) \in \operatorname{dom}(val(m) \Rightarrow_a)$.

Let us consider V, A, inp, and d. Let val be a finite sequence. We say that inp is a valid input of V, A, val and d if and only if

(Def. 10) there exists a non-atomic nominative data d_1 of V and A such that $d = d_1$ and val is valid w.r.t. d_1 and for every natural number n such that $1 \leqslant n \leqslant \operatorname{len} inp$ holds $d_1(val(n)) = inp(n)$.

The functor $\mathrm{ValInp}(V, A, val, inp)$ yielding a partial predicate over simple-named complex-valued nominative data of V and A is defined by

(Def. 11) $\operatorname{dom} it = \mathrm{ND}_{\mathrm{SC}}(V, A)$ and for every object d such that $d \in \operatorname{dom} it$ holds if inp is a valid input of V, A, val and d, then $it(d) = true$ and if inp is not a valid input of V, A, val and d, then $it(d) = false$.

Observe that $\mathrm{ValInp}(V, A, val, inp)$ is total.

Let us consider d. Let Z, res be finite sequences. We say that res is a valid output of V, A, Z and d if and only if

(Def. 12) there exists a non-atomic nominative data d_1 of V and A such that $d = d_1$ and Z is valid w.r.t. d_1 and for every natural number n such that $1 \leqslant n \leqslant \operatorname{len} Z$ holds $d_1(Z(n)) = res(n)$.

Let Z, res be objects. The functor $\mathrm{ValOut}(V, A, Z, res)$ yielding a partial predicate over simple-named complex-valued nominative data of V and A is defined by

(Def. 13) $\operatorname{dom} it = \{d$, where d is a nominative data with simple names from V and complex values from $A : d \in \operatorname{dom}(Z \Rightarrow_a)\}$ and for every object d such that $d \in \operatorname{dom} it$ holds if $\langle res \rangle$ is a valid output of V, A, $\langle Z \rangle$ and d, then $it(d) = true$ and if $\langle res \rangle$ is not a valid output of V, A, $\langle Z \rangle$ and d, then $it(d) = false$.

Now we state the propositions:

(16) Let us consider a z_2-element finite sequence val. Suppose loc, val and z_2 are correct w.r.t. d_1 and $d = (\mathrm{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - 1)$ and $2 \leqslant n + 1 < z_2$ and $d \nabla_a^{(loc/ \operatorname{len} val)} (val(\operatorname{len} val) \Rightarrow_a)(d) \in \operatorname{dom} p$. Then $(\mathrm{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - n - 1) \nabla_a^{(loc/ \operatorname{len} val - n)} (val(\operatorname{len} val - n) \Rightarrow_a)((\mathrm{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - n - 1)) \in \operatorname{dom}((\mathrm{ScPsuperposSeq}(loc, val, p))(n))$.

PROOF: Set $S = \mathrm{ScPsuperposSeq}(loc, val, p)$. Set $L = \mathrm{LocalOverlapSeq}(A, loc, val, d_1, z_2)$. Define $\mathcal{F}(\text{natural number}) = L(z_2 - \$_1 - 1) \nabla_a^{(loc/ \operatorname{len} val - \$_1)} (val(\operatorname{len} val - \$_1) \Rightarrow_a)(L(z_2 - \$_1 - 1))$. Define $\mathcal{P}[\text{natural number}] \equiv$ if $2 \leqslant \$_1 + 1 < z_2$, then $\mathcal{F}(\$_1) \in \operatorname{dom}(S(\$_1))$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k + 1]$. For every natural number k, $\mathcal{P}[k]$. □

(17) Let us consider a z_2-element finite sequence val. Suppose loc, val and z_2 are correct w.r.t. d_1 and $d = (\mathrm{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 -$

1) and $d\nabla_a^{(loc/_{\text{len}\, val})}(val(\text{len}\, val) \Rightarrow_a)(d) \in \text{dom}\, p$. Let us consider natural numbers m, n. Suppose $1 \leqslant m < z_2$ and $1 \leqslant n < z_2$. Then $((\text{ScPsuperposSeq}(loc, val, p))(m))((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - m)) = (\text{ScPsuperposSeq}(loc, val, p))(n)((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - n))$.

Proof: Set $S = \text{ScPsuperposSeq}(loc, val, p)$. Set $L = \text{LocalOverlapSeq}(A, loc, val, d_1, z_2)$. Define $\mathcal{P}[\text{natural number}] \equiv$ if $1 \leqslant \$_1 < z_2$, then $(S(m))(L(z_2 - m)) = S(\$_1)(L(z_2 - \$_1))$. For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. For every natural number k, $\mathcal{P}[k]$. □

(18) Let us consider a z_2-element finite sequence val, objects d_4, d_5, and a natural number N_1. Suppose $N_1 = z_2 - 2$. Suppose loc, val and z_2 are correct w.r.t. d_1 and $d_4 = (\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - 1)$ and $d_4\nabla_a^{(loc/_{\text{len}\, val})}(val(\text{len}\, val) \Rightarrow_a)(d_4) \in \text{dom}\, p$ and $d_5 = (\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(N_1)\nabla_a^{(loc/_{N_1+1})}(val(N_1+1) \Rightarrow_a)((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(N_1))$ and $d_5\nabla_a^{(loc/_{\text{len}\, val})}(val(\text{len}\, val) \Rightarrow_a)(d_5) \in \text{dom}\, p$. Then $((\text{ScPsuperposSeq}(loc, val, p))(1))((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - 1)) = p((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2))$. The theorem is a consequence of (15).

(19) Let us consider a z_2-element finite sequence val, and a partial predicate over simple-named complex-valued nominative data p of V and A. Suppose $3 \leqslant z_2$ and loc, val and z_2 are correct w.r.t. d_1 and $(\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - 1)\nabla_a^{(loc/_{\text{len}\, val})}(val(\text{len}\, val) \Rightarrow_a)((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(z_2 - 1)) \in \text{dom}\, p$ and $d_1\nabla_a^{(loc/_1)}(val(1) \Rightarrow_a)(d_1) \in \text{dom}((\text{ScPsuperposSeq}(loc, val, p))(z_2 - 1))$. Then $((\text{ScPsuperposSeq}(loc, val, p))(\text{len}\, \text{ScPsuperposSeq}(loc, val, p)))(d_1) = (\text{S}_\text{P}((\text{ScPsuperposSeq}(loc, val, p))(z_2 - 2), val(2) \Rightarrow_a, loc/_2))((\text{LocalOverlapSeq}(A, loc, val, d_1, z_2))(1))$. The theorem is a consequence of (16) and (17).

3. Sequences of Local Overlappings

Let us consider V, A, loc, d_1, and pos. Let prg be a $(\text{FPrg}(\text{ND}_{\text{SC}}(V, A)))$-valued finite sequence. Assume $\text{len}\, prg > 0$. The functor PrgLocOverlapSeq(A, loc, d_1, prg, pos) yielding a finite sequence of elements of $\text{ND}_{\text{SC}}(V, A)$ is defined by

(Def. 14) $\text{len}\, it = \text{len}\, prg$ and $it(1) = d_1\nabla_a^{(loc/_{pos(1)})}prg(1)(d_1)$ and for every natural number n such that:
$1 \leqslant n < \text{len}\, it$ holds $it(n+1) = it(n)\nabla_a^{(loc/_{pos(n+1)})}prg(n+1)(it(n))$.

Let us consider prg. Note that PrgLocOverlapSeq(A, loc, d_1, prg, pos) is (V,A)-nonatomicND yielding.

Let us consider n. One can verify that (PrgLocOverlapSeq(A, loc, d_1, prg, pos)) (n) is function-like and relation-like.

We say that prg is domain closed w.r.t. loc, d_1 and pos if and only if

(Def. 15) for every natural number n such that $1 \leqslant n < \operatorname{len} prg$ holds (PrgLocOverlapSeq(A, loc, d_1, prg, pos))(n) $\in$ dom($prg(n + 1)$).

Now we state the proposition:

(20) Suppose $1 \leqslant n \leqslant \operatorname{len} prg$ and (PrgLocOverlapSeq(A, loc, d_1, prg, pos))(m) $\in$ dom($prg(n)$). Then $prg(n)$((PrgLocOverlapSeq(A, loc, d_1, prg, pos))(m)) is a nominative data with simple names from V and complex values from A.

Let us consider a natural number n. Now we state the propositions:

(21) Suppose V is not empty and V is without nonatomic nominative data w.r.t. A. Then suppose $1 \leqslant n < \operatorname{len} prg$ and (PrgLocOverlapSeq(A, loc, d_1, prg, pos))(n) $\in$ dom($prg(n+1)$). Then dom((PrgLocOverlapSeq(A, loc, d_1, prg, pos))($n+1$)) $= \{loc_{/pos(n+1)}\} \cup$dom((PrgLocOverlapSeq($A, loc, d_1, prg, pos$))($n$)). The theorem is a consequence of (20).

(22) Suppose V is not empty and V is without nonatomic nominative data w.r.t. A. Then suppose $1 \leqslant n < \operatorname{len} prg$ and (PrgLocOverlapSeq(A, loc, d_1, prg, pos))(n) $\in$ dom($prg(n+1)$). Then dom((PrgLocOverlapSeq(A, loc, d_1, prg, pos))(n)) $\subseteq$ dom((PrgLocOverlapSeq(A, loc, d_1, prg, pos))($n+1$)). The theorem is a consequence of (21).

(23) Suppose V is not empty and V is without nonatomic nominative data w.r.t. A and dom(PrgLocOverlapSeq(A, loc, d_1, prg, pos)) $\subseteq$ dom prg and $d_1 \in$ dom($prg(1)$) and prg is domain closed w.r.t. loc, d_1 and pos. Then if $1 \leqslant n \leqslant \operatorname{len} prg$, then dom $d_1 \subseteq$ dom((PrgLocOverlapSeq(A, loc, d_1, prg, pos))(n)).

PROOF: Set $F =$ PrgLocOverlapSeq(A, loc, d_1, prg, pos). Define $\mathcal{P}$[natural number] $\equiv$ if $1 \leqslant \$_1 \leqslant \operatorname{len} prg$, then dom $d_1 \subseteq$ dom($F(\$_1)$). For every natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$. For every natural number k, $\mathcal{P}[k]$. □

Let us consider natural numbers m, n. Now we state the propositions:

(24) Suppose V is not empty and V is without nonatomic nominative data w.r.t. A and prg is domain closed w.r.t. loc, d_1 and pos. Then suppose $1 \leqslant n \leqslant m \leqslant \operatorname{len} prg$. Then dom((PrgLocOverlapSeq(A, loc, d_1, prg, pos))(n)) $\subseteq$ dom((PrgLocOverlapSeq(A, loc, d_1, prg, pos))(m)). The theorem is a consequence of (22).

(25) Suppose V is not empty and V is without nonatomic nominative data w.r.t. A and dom(PrgLocOverlapSeq(A, loc, d_1, prg, pos)) $\subseteq$ dom prg and $d_1 \in$ dom($prg(1)$) and prg is domain closed w.r.t. loc, d_1 and pos. Then if

$1 \leqslant n \leqslant m \leqslant \operatorname{len} prg$, then $loc_{/pos(n)} \in \operatorname{dom}((\text{PrgLocOverlapSeq}(A, loc, d_1, prg, pos))(m))$. The theorem is a consequence of (24).

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Accepted October 25, 2020